Government of Karnataka

GANITA PRAKASH

Textbook of Mathematics

6

SIXTH STANDARD

Part - 1

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एन सी ई आर टी NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

Karnataka Textbook Society (R.)
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Foreword

The National Education Policy (NEP) 2020 envisages a system of education in the country that is rooted in Indian ethos and its civilisational accomplishments in all domains of human endeavour and knowledge, while at the same time preparing the students to constructively engage with the prospects and challenges of the twenty-first century. The basis for this aspirational vision has been well laid out by the National Curriculum Framework for School Education (NCFSE) 2023 across curricular areas at all stages. Nurturing the students' inherent abilities touching upon all the five planes of human existence, the *pañchakośhas*, in the Foundational and the Preparatory Stages has paved the way for the progression of their learning further at the Middle Stage. The Middle Stage acts as a bridge between the Preparatory and the Secondary Stages, spanning three years from Grade 6 to 8.

This framework, at the Middle Stage, aims to equip students with the skills that are needed to grow as they advance in their lives. It endeavours to enhance their analytical, descriptive, and narrative capabilities, and to prepare them for the challenges and opportunities that await them. A diverse curriculum, covering nine subjects ranging from three languages—including at least two languages native to India—to Science, Mathematics, Social Sciences, Art Education, Physical Education and Well-being, and Vocational Education promotes their holistic development.

Such a transformative learning culture requires certain essential conditions. One of them is to have appropriate textbooks in different curricular areas, as these textbooks will play a central role in mediating between content and pedagogy—a role that will strike a judicious balance between direct instruction and opportunities for exploration and inquiry. Among other conditions, classroom arrangement and teacher preparation are crucial to establish conceptual connections both within and across curricular areas.

The National Council of Educational Research and Training (NCERT), in its part, is committed to provide students with such high-quality textbooks. Various Curricular Area Groups, which have been constituted for this purpose, comprising notable subject-experts,

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pedagogues, and practising teachers as their members, have made all possible efforts to develop such textbooks. *Ganita Prakash*, the textbook of Mathematics for Grade 6, is one of these. The textbook is a captivating journey through the world of mathematics designed for Grade 6 students. The book begins with encouraging the students to observe and explore the patterns around them and discover mathematical concepts on their own. The book further delves into the realm of numbers, where young learners are introduced to the magic of numbers and shapes. Through colourful illustrations and interactive exercises, children develop a strong foundation in arithmetic, paving the way for more complex mathematical concepts. Throughout the book, stories, conversations and anecdotes have been incorporated to make abstract mathematical concepts more relatable and accessible to young learners. Content has been evolved using puzzles and innovative problems that will not only engage the students in thoughtfully relating the mathematical concepts to the world around them and help them in deepening their understanding of mathematics, but also prepare them to understand the concepts of the emerging field of Computational Thinking. Indian rootedness and relation to Indian Knowledge Systems (IKS) has been embedded in the content of the textbook.

However, in addition to this textbook, students at this stage should also be encouraged to explore various other learning resources. School libraries play a crucial role in making such resources available. Besides, the role of parents and teachers will also be invaluable in guiding and encouraging students to do so.

With this, I express my gratitude to all those who have been involved in the development of this textbook and hope that it will meet the expectations of all stakeholders. At the same time, I also invite suggestions and feedback from all its users for further improvement in the coming years.

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Dinesh Prasad Saklani
Director
National Council of Educational
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New Delhi
July, 2024

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About the Book

Mathematics helps students develop not only basic arithmetic skills, but also the crucial capacities of logical reasoning, creative problem solving, and clear and precise communication (both oral and written). Mathematical knowledge also plays a crucial role in understanding concepts in other school subjects, such as Science and Social Science, and even Art, Physical Education, and Vocational Education. Learning Mathematics can also contribute to the development of capacities for making informed choices and decisions. Understanding numbers and quantitative arguments is necessary for effective and meaningful democratic and economic participation. Mathematics thus has an important role to play in achieving the overall Aims of School Education.

Mathematics at the Middle Stage is a major challenge and has to perform the dual role of being both close to the experience and environment of the child and being abstract. It must perform the dual role of developing intuition while also maintaining and emphasising rigour. It must perform the dual role of enhancing critical and logical thinking while also developing artistry and creativity and a sense of elegance and aesthetics. Finally, Mathematics must perform the dual role of providing students plenty of opportunities for exploration and discovery of concepts on their own while also teaching best-known methods in the global repertoire of mathematics.

The present textbook has made an attempt to address the above-mentioned goals and challenges of learning mathematics. The writers of this book have aimed to strike a judicious balance between informal and formal definitions and methods to develop in students both intuition and rigour. The book also provides numerous opportunities for student–student and student–teacher interaction in the classroom to promote active and experiential learning. A number of questions, puzzles, and interactive exercises are posed throughout the book to encourage constant exploration. Many of the questions are open-ended to stimulate in-class discussion. Finally, some famous unsolved problems have also been included so that students can appreciate that Mathematics is still a very active subject, with much that is already known and discovered, but also many exciting frontiers that remain unknown and unseen. Such unknown realms and unresolved questions will require new ideas and a new generation of adventurers to explore and understand, and to thereby solve these exciting problems.

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Among the world's greatest problem solvers and most creative minds of the current generation is world-renowned mathematician Manjul Bhargava. He has resolved decades-old, and in some cases centuries-old, problems of a fundamental nature across Mathematics, particularly in the areas of number theory, algebra, representation theory, and arithmetic geometry. For his pioneering breakthroughs in Mathematics, in 2014 he became the first person of Indian origin to receive the Fields Medal, the highest honour given to mathematicians, awarded every four years and known as the 'Nobel Prize of Mathematics'.

We are thrilled and honoured that the beautiful Chapter 1 of this book, 'Patterns in Mathematics', has been kindly composed and contributed by Professor Bhargava. In this chapter, in the section 'What is Mathematics?', Bhargava eloquently speaks of mathematics as a creative art—as a search for beautiful patterns, and the explanations of those patterns. In later sections of the chapter, he describes a sampling of some of the most basic patterns in mathematics—sequences of numbers and sequences of shapes—and their remarkable and often-surprising interrelations. These patterns are regularly revisited in later chapters of this book, to emphasise the unity of mathematics, and will also be revisited in future years. We hope that this exploratory chapter will help in inspiring a new generation to explore and pursue mathematics.

Building on the idea of exploring patterns in mathematics, the book then turns to a journey across different areas of mathematics. Chapter 2, 'Lines and Angles', introduces the building blocks of geometry—points, line segments, rays, lines, angles, and how to measure angles. Chapter 3, 'Number Play', is an exploratory adventure through some instructive but fun games and puzzles in mathematics—some of which are still unsolved! Chapter 4, 'Data Handling', is an introduction to the art of collecting and presenting data, including both its analytic and aesthetic aspects. Chapter 5, 'Prime Time', is a playful adventure through prime numbers—the building blocks of the universe of whole numbers—and factorization. Chapter 6, 'Perimeter and Area', is a revision of these fundamental notions, with a variety of challenging puzzles to keep children on their toes and enhance understanding. Chapter 7, 'Fractions', will be many students' first encounter with this important concept; the chapter aims to build intuition about fractions gradually, starting with fractional units like 1/10 as the foundation, and gradually building up to working with general fractions, including

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their comparison, addition, and subtraction. Chapter 8, 'Playing with Constructions', is a hands-on experience of drawing shapes, including using a compass and a ruler, to enhance students' geometric intuition and comprehension. Chapter 9, 'Symmetry', is an artistic and hands-on exploration of this most important and ubiquitous concept in Mathematics and beyond. Finally, Chapter 10, 'The Other Side of Zero', aims for students to gain intuition for negative numbers by visiting Bela's Building of Fun, and gradually working up to understanding the laws of addition and subtraction of all integers as laid down by Brahmagupta.

In all chapters, an attempt has been made to emphasise connections with other subjects including Art, History, and Science. Many pictures and drawings have been included to illustrate patterns, numbers, constructions, symmetry, games, puzzles, etc., to thereby develop visual and artistic imagination, intuition for mathematical objects and principles. The history of various mathematical concepts has been described, including Brahmagupta's world-changing discoveries in the year 628 C.E. of the laws for addition and subtraction of fractions and of zero and negative numbers. Other discoveries from around the world, of unit fractions, searching for primes, Collatz Conjecture, Kaprekar numbers, etc., have also been described with their history to help students appreciate and humanize the joy and process of discovery. Examples from Science (the use of negative numbers to measure temperature or heights above or below sea level) also abound to illustrate the importance of the use of mathematical concepts in Science.

By weaving together storytelling and hands-on activities, we hope that an immersive learning experience will be created that ignites curiosity and fosters a love for mathematics. It is hoped that teachers would give children the opportunity to discuss, play, engage with each other, provide logical arguments for different ideas, and find loopholes in arguments presented. This is necessary for the learners to eventually develop the ability to understand what it means to prove something and also become confident about underlying concepts. The mathematics classroom should not expect a blind application of algorithms but should rather encourage children to find many different ways to solve problems.

As per the NEP 2020, Computational Thinking has also been gently introduced through puzzles, games, and interactive exercises that encourage such thinking. Indian rootedness has also been kept in

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mind while giving contexts for different concepts. The contributions of Indian mathematicians have been given as part of a problem-solving approach to make students aware of India's rich mathematical heritage and its global contributions to mathematics.

The concepts and problems are related to daily life situations. An attempt has been made to use contexts and materials with which the students are familiar. Learning material sheets have been given at the back of the book that may be photocopied and used. At many places, exercises or activities are given to encourage peer group efforts and discussions. The textbook intends to address the learning needs of a diverse group of students in the classroom.

We have tried to link concepts learnt in initial chapters with ideas in subsequent chapters to show the connectedness and unity of mathematics. We hope that teachers will use this as an opportunity to revise these concepts in a spiralling way so that children are able to appreciate the entire conceptual structure of mathematics. We hope that teachers may give more time to the ideas of fractions, negative numbers and other notions that are new to students. Many of these are the basis for further learning in mathematics.

Finally, this book aims to be more than just a textbook—it's a passport to a world of mathematical discovery and exploration. Whether used in the classroom or at home, we hope that it may inspire students to embark on their own mathematical adventures, empowering them to see the beauty and relevance of mathematics in everything around them. With its engaging approach and comprehensive coverage of Grade 6 mathematics concepts, this book hopes and aims to captivate young minds and set them on a lifelong journey of mathematical discovery.

I thank again all the writers of and contributors to this textbook for this important and valuable contribution and service to the nation's mathematics teachers, learners and enthusiasts.

We look forward to your comments and suggestions regarding the book and hope that you will send interesting exercises, activities and tasks that you develop during the course of teaching and learning, to be included in future editions.

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Note to the Teacher

We hope that this book, Ganita Prakash, will serve as a strong support and guide to you in achieving the exciting task that you have before you: that of passing on the joy of learning the beautiful subject of mathematics to the next generation.

This task calls for providing a fertile environment that allows for the flowering of mathematical thinking in the minds of students.

Classrooms, where students just listen and write down whatever is being told to them or written on the board, are deficient in the conditions required for learning mathematics. Instead, classrooms need to be places where students are engaged in playing with mathematical concepts, finding and discussing patterns, and developing creative strategies together to solve problems. Students should also be posing problems to each other and discussing possible solutions with each other. In fact, these are the very conditions that have led to the development of the entire field of mathematics so far, and so one cannot expect students to pick up mathematical thinking and understanding without these conditions.

Fortunately, it is not difficult to create such conditions in the classroom. It just requires an interesting question, problem, pattern, or challenge to be thrown open to the students on a regular basis, and sufficient time to be given to them to play with, discuss, and work on it as a class or in pairs or groups.

Along with it, an environment that accepts mistakes and acknowledges their importance in learning needs to be nurtured.

While creating the spark for initiating mathematical thinking in classrooms is not difficult, sustaining it may be challenging and may involve efforts from your side. Nevertheless, even if just the first part of throwing open a question, problem, pattern, or challenge is done at least once or twice a week, accompanied by sufficient waiting time from your side for students to play, discuss, and work on it, it can have a great positive impact on how the students view and approach mathematics.

It should be noted that this positive impact will not happen overnight. That takes time and depends on various factors such as the number of opportunities you give for problem solving, your patience, and the encouragement you give to the students.

To support you in posing problems, all the problems or questions in this book are marked using the icon  . This icon is an indicator of a potential opportunity to start off a process of problem solving and exploration in the classroom. You will find some of the problems labelled ‘Math Talk’. Such questions can especially be made as topics for classroom discussion.

To develop students’ mathematical thinking and understanding of concepts, a sufficient number of problems are given. Trying to ‘cover’ all of them must not happen at the cost of students not getting to spend quality time on playing with and discussing them.

It is important to understand that the exploratory problems are not only for promoting problem solving skills; they also serve in strengthening procedural fluency when children start engaging in exploration.

Efforts must be made in making students independent learners. One essential aspect required for this is an ability to read and understand mathematical text. To promote this

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skill, students should be encouraged to read the book by themselves and in groups. Give opportunities to them to interpret what they read and express it to others. This will also address the big problem that students face in speaking mathematics and interpreting word problems.

This book contains a number of open-ended problems. It also contains new treatments of certain concepts. If you are not able to solve them or follow some of them immediately, it is perfectly okay! Not everyone knows everything. Along with trying to understand and reflect upon such content, it will be very useful to take it to the classroom and open it up for discussion. After the discussion, things that are clear and those that are not yet clear can be clearly summarised. This process itself can throw a lot of light on the content. In these discussions, you can participate as a fellow seeker, and when students see a teacher seek and think to understand something, it sets a wonderful example for them.

It is hoped that you and your students will have a great and fruitful time using this book!

Summary of Key points

Time for Exploration

1. It is important to routinely pose new problems, questions, patterns, or challenges to the students and give them sufficient time to play with, discuss, and work on them, individually and in groups.
2. During this time, an environment that accepts mistakes and acknowledges their importance in learning needs to be nurtured.
3. There should be a culture where students pose problems to each other and discuss with each other various ways to approach the problems.

About the Problems in the Book

1. The exploratory problems in the book not only promote problem solving; they also aim to strengthen procedural fluency when children start engaging in exploration.
2. Trying to 'cover' all the problems in the book must not happen at the cost of students not getting to spend quality time on playing with, discussing, and solving them.

Reading

1. Encourage students to read the book by themselves and in groups.
2. Give opportunities to them to interpret what they read and to express it to others.

Right of Not Knowing!

1. It is perfectly okay if some of the content is not understood immediately. Along with trying to understand and reflect upon such content, it can also be taken to the classroom and opened up for discussion. After the discussion, things that are clear and those that are not yet clear can be clearly summarised. In these discussions, you can participate as a fellow seeker, and when students see a teacher seek and think to understand something, it sets a wonderful example for them!
2. Learning is a continual process. Indeed, there is so much in mathematics that is still not known and requires further exploration!

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A Note to Students!

To be able to appreciate the art of mathematics, it is not enough to just be a passive spectator. You need to immerse yourself in its process like a detective getting into action to solve a mystery.

This is especially required when you see a new question or when a question arises from your own sense of wonder, or when you come across a new beautiful pattern. When you encounter these, pause your reading, and use your creativity to work out the question or understand and appreciate the pattern.

You will find that some questions are accompanied by their answers. Even if this is the case, it is worthwhile to work on the problems by yourself or in a group before you see the answer. This will enrich your experience of going through the book!

Whenever there are questions coming up, you will see this icon: **.** This indicates that it is time for figuring things out! Sometimes you will find many questions collected together in a single place under the title '**Figure it Out**'.

Some questions are marked 

.These questions are meant to be discussed and worked out with your friends.

Finally, there are questions marked 

. These questions demand more creativity to be answered, and therefore will also often be more fun to answer as a result!

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ಮುನ್ನುಡಿ

2025–26 ನೇ ಶೈಕ್ಷಣಿಕ ಸಾಲಿನಿಂದ ರಾಜ್ಯದ ಎಲ್ಲಾ 6ನೇ ತರಗತಿ ವಿದ್ಯಾರ್ಥಿಗಳ ಕಲಿಕೆಗಾಗಿ ಲಭ್ಯವಾಗುತ್ತಿರುವ 'ಗಣಿತ ಪ್ರಕಾಶ' ಪಠ್ಯಪುಸ್ತಕವನ್ನು ಎನ್.ಸಿ.ಆರ್.ಟಿ ಯು 2020 ರ ರಾಷ್ಟ್ರೀಯ ಶಿಕ್ಷಣ ನೀತಿ ಹಾಗೂ 2023 ರ ರಾಷ್ಟ್ರೀಯ ಪಠ್ಯಕ್ರಮ ಚೌಕಟ್ಟು (ಶಾಲಾ ಶಿಕ್ಷಣ)ಗಳ ಆಶಯಗಳನ್ನು ಒಳಗೊಂಡು ರಚಿಸಿದೆ. ವಿದ್ಯಾರ್ಥಿಗಳ ಅಂತರ್ಗತ ಸಾಮರ್ಥ್ಯಗಳ ಅಭಿವೃದ್ಧಿಯೊಂದಿಗೆ 21 ನೇ ಶತಮಾನದ ಸವಾಲುಗಳನ್ನು ಎದುರಿಸಲು ಸಮರ್ಥರಾಗುವಂತೆ ವಿದ್ಯಾರ್ಥಿಗಳನ್ನು ಸಿದ್ಧಪಡಿಸಬೇಕಿರುವುದು ಮಾತ್ರವಲ್ಲ, ಅವರ ಭವಿಷ್ಯದ ಬೆಳೆವಣಿಗೆಗೆ ಅಗತ್ಯವಿರುವ ಕೌಶಲಗಳನ್ನು ಗಳಿಸಿಕೊಳ್ಳುವಂತೆ ಮಾಡುವುದು ತುಂಬಾ ಅಗತ್ಯವೆನಿಸುತ್ತದೆ. ಸವಾಲುಗಳನ್ನು ಎದುರಿಸಿ, ಅವಕಾಶಗಳನ್ನು ಸದ್ಬಳಕೆ ಮಾಡಿಕೊಳ್ಳಲು ಸಿದ್ಧರಿರುವಂತೆ ಅವರ ವಿಶ್ಲೇಷಣಾತ್ಮಕ, ವಿವರಣಾತ್ಮಕ ಮತ್ತು ನಿರೂಪಣಾ ಸಾಮರ್ಥ್ಯಗಳನ್ನು ಪೋಷಿಸಬೇಕಿದೆ. ಈ ಹಿನ್ನೆಲೆಯಲ್ಲಿ 'ಗಣಿತ ಪ್ರಕಾಶ' ಪಠ್ಯಪುಸ್ತಕ ಮಹತ್ತರವಾದ ಪಾತ್ರವನ್ನು ವಹಿಸುತ್ತದೆ ಎಂದರೆ ತಪ್ಪಾಗಲಾರದು.

ಪಠ್ಯವಸ್ತು ಹಾಗೂ ಬೋಧನಾ ಶಾಸ್ತ್ರಗಳು ಸೂಕ್ತವಾಗಿ ಸಮನ್ವಯಗೊಂಡಿರುವ ಈ ಪಠ್ಯಪುಸ್ತಕದಲ್ಲಿ ವಿದ್ಯಾರ್ಥಿಗಳ ಕುತೂಹಲಕ್ಕೆ ನೀರೆರೆಯುತ್ತಾ, ಅವರ ವಿಚಾರಣಾ ಸಾಮರ್ಥ್ಯವನ್ನು ವೃದ್ಧಿಸಿ, ಅನ್ವೇಷಣಾ ಮಾರ್ಗದಲ್ಲಿ ಜ್ಞಾನ ಕಟ್ಟಿಕೊಳ್ಳಲು ಹೇರಳ ಅವಕಾಶವನ್ನು ನೀಡಲಾಗಿದೆ. ವಿದ್ಯಾರ್ಥಿ ತನ್ನ ಸುತ್ತಮುತ್ತಲಿನ ಪ್ರಪಂಚವನ್ನು ಗಮನಿಸಿ, ಶೋಧಿಸಿ ಅಲ್ಲಿ ಅನುಭವಿಸುವ ಗಣಿತೀಯ ಪರಿಕಲ್ಪನೆಗಳನ್ನು ಆರಂಭದಲ್ಲಿಯೇ ಪರಿಚಯಿಸಲಾಗಿದೆ. ಇಲ್ಲಿ ಬಳಕೆಯಾಗಿರುವ ವರ್ಣರಂಜಿತ ಉದಾಹರಣೆಗಳು, ಸಂವಾದಾತ್ಮಕ ಅಭ್ಯಾಸಗಳು ವಿದ್ಯಾರ್ಥಿಗಳ ಲೆಕ್ಕಾಚಾರದ ಬುನಾದಿಯನ್ನು ಬಲಿಷ್ಠಗೊಳಿಸಿ ಸಂಕೀರ್ಣ ಗಣಿತೀಯ ಪರಿಕಲ್ಪನೆಗಳ ಕಲಿಕೆಯನ್ನು ಸರಳಗೊಳಿಸುತ್ತವೆ. ಪರಿಕಲ್ಪನೆಗಳನ್ನು ಅರ್ಥೈಸಲು ಪಠ್ಯಗಳಲ್ಲಿ ಅಳವಡಿಸಿಕೊಂಡಿರುವ ಕಥೆಗಳು, ಸಂವಾದಗಳು ಕಲಿಕಾರ್ಥಿಗಳ ಸಹಜ, ಸರಳ ಕಲಿಕೆಗೆ ಪೂರಕವಾಗಿವೆ. ಅಭ್ಯಾಸಕ್ಕಾಗಿ ನಾವೀನ್ಯ ಸಮಸ್ಯೆಗಳು ಹಾಗೂ ಒಗಟುಗಳನ್ನು ಪರಿಣಾಮಕಾರಿಯಾಗಿ ಬಳಸಿಕೊಂಡಿರುವುದರಿಂದ ಕಲಿಕಾರ್ಥಿಗಳು ಗಮನವನ್ನು ಕಲಿಕೆಯ ಮೇಲೆ ಕೇಂದ್ರೀಕರಿಸಲು ಹಾಗೂ ಗಣಿತೀಯ ಪರಿಕಲ್ಪನೆಗಳನ್ನು ಆಳವಾಗಿ ಅರ್ಥೈಯಿಸಿಕೊಂಡು ಅನ್ವಯಮಾಡಿಕೊಳ್ಳಲು ಸಹಾಯ ಮಾಡುತ್ತದೆ. ಈ ಮೂಲಕ ತಮ್ಮ ಗಣಿತೀಯ ಅಲೋಚನಾ (Computational Thinking) ಸಾಮರ್ಥ್ಯವನ್ನು ವೃದ್ಧಿಸಿಕೊಳ್ಳಲು ಪ್ರೇರಣೆ ಒದಗಿಸುತ್ತದೆ.

ನೀಡಿರುವ ಪರಿಕಲ್ಪನೆಗಳು ಹಾಗೂ ಬಳಸಿರುವ ಸಮಸ್ಯೆಗಳು ವಿದ್ಯಾರ್ಥಿಗಳ ನಿತ್ಯಜೀವನಕ್ಕೆ ಸಂಬಂಧಿಸಿರುವುದರಿಂದ ಪರಿಕಲ್ಪನೆಗಳ ಅನ್ವಯ ವಿದ್ಯಾರ್ಥಿಗಳಿಗೆ ಸರಳವಾಗುತ್ತದೆ. ಎಲ್ಲಾ ಅಧ್ಯಾಯಗಳಲ್ಲೂ ಕಲೆ, ಇತಿಹಾಸ, ವಿಜ್ಞಾನ ಇತ್ಯಾದಿ ವಿಷಯಗಳನ್ನು ಸಮ್ಮಿಳಿತಗೊಳಿಸಿರುವುದರಿಂದ ಬಹುಶಿಸ್ತೀಯ ಕಲಿಕೆಗೆ ಅನುಕೂಲವಾಗುತ್ತದೆ. ಪರಿಕಲ್ಪನೆಗಳನ್ನು ಕಥನ ಶೈಲಿಯಲ್ಲಿ ಹಾಗೂ ಮಾಡಿ ಕಲಿ ತತ್ತ್ವದಡಿಯಲ್ಲಿ ಕಲಿಸಲು ಅವಕಾಶ ನೀಡಿರುವುದರಿಂದ ವಿದ್ಯಾರ್ಥಿಗಳಲ್ಲಿ ಗಣಿತದ ಬಗ್ಗೆ ಕುತೂಹಲ ಮತ್ತು ಪ್ರೀತಿ ಹೆಚ್ಚುವುದರಲ್ಲಿ ಅನುಮಾನವಿಲ್ಲ.

ಇಷ್ಟೆಲ್ಲ ವಿಶೇಷ ಗುಣಲಕ್ಷಣಗಳನ್ನು ಹೊಂದಿರುವ 'ಗಣಿತ ಪ್ರಕಾಶ' ಪುಸ್ತಕವನ್ನು ತರಗತಿ ಪ್ರಕ್ರಿಯೆಯಲ್ಲಿ ಅಳವಡಿಸಿಕೊಳ್ಳಲು ಸುಗಮಕಾರರು ತಮ್ಮ ತರಗತಿಯನ್ನು ಅದಕ್ಕೆ ಪೂರಕವಾಗುವಂತೆ ವ್ಯವಸ್ಥೆಗೊಳಿಸಿಕೊಳ್ಳುವುದು ಹಾಗೂ ಪೂರ್ವಭಾವಿ ಸಿದ್ಧತೆಗಳೊಂದಿಗೆ ತರಗತಿ ನಿರ್ವಹಣೆ ಮಾಡುವುದು

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ತುಂಬಾ ಅಗತ್ಯವೆನಿಸುತ್ತದೆ. ಕುರುಡು ಲೆಕ್ಕಾಚರದ ಬದಲಿಗೆ ಒಗಟು, ಚರ್ಚೆ, ಸಂವಾದ, ಆಟ, ಪರಸ್ಪರ ತೊಡಗಿಸಿಕೊಳ್ಳುವಿಕೆ, ತಾರ್ಕಿಕವಾಗಿ ಆಲೋಚಿಸಲು ಸಾಧ್ಯವಾಗುವಂತಹ ಚಟುವಟಿಕೆಗಳನ್ನು ಒಳಗೊಂಡ ಸುಗಮಗಾರಿಕೆಯನ್ನು ಈ ಪಠ್ಯಪುಸ್ತಕ ನಿರೀಕ್ಷೆ ಮಾಡುತ್ತದೆ. ಈ ನಿರೀಕ್ಷೆಯನ್ನು ನಮ್ಮ ಸುಗಮಕಾರರು ಹುಸಿ ಮಾಡುವುದಿಲ್ಲ ಎಂಬ ನಂಬಿಕೆ ರಾಜ್ಯ ಶಿಕ್ಷಣ ಇಲಾಖೆಯದ್ದಾಗಿದೆ.

ಎನ್.ಸಿ.ಇ.ಆರ್.ಟಿ ಪ್ರಕಟಿಸಿರುವ ಈ ಪಠ್ಯಪುಸ್ತಕವನ್ನು ಕರ್ನಾಟಕ ರಾಜ್ಯದ ವಿದ್ಯಾರ್ಥಿಗಳು ರಾಷ್ಟ್ರಮಟ್ಟದ ಎಲ್ಲಾ ಅವಕಾಶಗಳನ್ನು ತಮ್ಮದಾಗಿಸಿಕೊಳ್ಳಬೇಕು ಎಂಬ ಆಶಯದೊಂದಿಗೆ 2025–26 ನೇ ಶೈಕ್ಷಣಿಕ ಸಾಲಿನಿಂದ ಅಳವಡಿಸಿಕೊಳ್ಳಲು ಅನುಮತಿ ನೀಡಿದ ಕರ್ನಾಟಕ ಸರ್ಕಾರದ ಶಿಕ್ಷಣ ಇಲಾಖೆಯ ಎಲ್ಲಾ ಹಂತದ ಅಧಿಕಾರಿಗಳಿಗೆ ಹಾಗೂ ಪಠ್ಯಪುಸ್ತಕ ಬಳಕೆಗೆ ಅವಕಾಶ ನೀಡಿದ ಎನ್.ಸಿ.ಇ.ಆರ್.ಟಿ ಯ ವಿವಿಧ ಹಂತದ ಅಧಿಕಾರಿಗಳಿಗೆ ಹಾಗೂ ಪ್ರಕಾಶನ ವಿಭಾಗಕ್ಕೆ ಹೃತ್ಪೂರ್ವಕ ಧನ್ಯವಾದಗಳು.

ಈ ಪಠ್ಯಪುಸ್ತಕವನ್ನು ಕನ್ನಡ, ಮರಾಠಿ, ತೆಲುಗು ಮತ್ತು ತಮಿಳು ಮಾಧ್ಯಮಗಳಿಗೆ ಅನುವಾದಿಸಿದ ಎಲ್ಲಾ ಸಂಪನ್ಮೂಲ ವ್ಯಕ್ತಿಗಳಿಗೆ, ಕಾರ್ಯಕ್ರಮ ಸಂಯೋಜನೆ ಮಾಡಿದ ಕಾರ್ಯಕ್ರಮಾಧಿಕಾರಿಗಳಿಗೆ, ಡಿ.ಟಿ.ಪಿ ಆಪರೇಟರ್‌ಗಳಿಗೆ ಹಾಗೂ ಮುದ್ರಕರಿಗೆ ಕರ್ನಾಟಕ ಪಠ್ಯಪುಸ್ತಕ ಸಂಘವು ಕೃತಜ್ಞತೆಗಳನ್ನು ಸಲ್ಲಿಸುತ್ತದೆ.

ಶ್ರೀ ಎಂ.ಪಿ. ಮಾದೇಗೌಡ
ವ್ಯವಸ್ಥಾಪಕ ನಿರ್ದೇಶಕರು
ಕರ್ನಾಟಕ ಪಠ್ಯಪುಸ್ತಕ ಸಂಘ (ರಿ.)
ಬೆಂಗಳೂರು.

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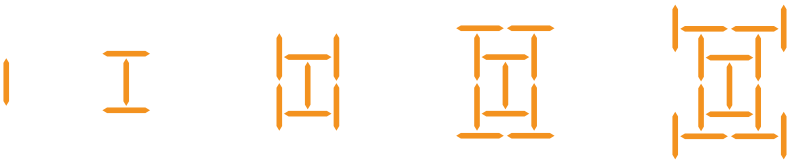
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1 PATTERNS IN MATHEMATICS

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1.1 What is Mathematics?

Mathematics is, in large part, the search for patterns, and for the explanations as to why those patterns exist.

Such patterns indeed exist all around us—in nature, in our homes and schools, and in the motion of the sun, moon, and stars. They occur in everything that we do and see, from shopping and cooking, to throwing a ball and playing games, to understanding weather patterns and using technology.

The search for patterns and their explanations can be a fun and creative endeavour. It is for this reason that mathematicians think of mathematics both as an art and as a science. This year, we hope that you will get a chance to see the creativity and artistry involved in discovering and understanding mathematical patterns.

It is important to keep in mind that mathematics aims to not just find out what patterns exist, but also the explanations for why they exist. Such explanations can often then be used in applications well beyond the context in which they were discovered, which can then help to propel humanity forward.

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For example, the understanding of patterns in the motion of stars, planets, and their satellites led humankind to develop the theory of gravitation, allowing us to launch our own satellites and send rockets to the Moon and to Mars; similarly, understanding patterns in genomes has helped in diagnosing and curing diseases—among thousands of other such examples.

Figure it Out



1. Can you think of other examples where mathematics helps us in our everyday lives?
2. How has mathematics helped propel humanity forward? (You might think of examples involving: carrying out scientific experiments; running our economy and democracy; building bridges, houses or other complex structures; making TVs, mobile phones, computers, bicycles, trains, cars, planes, calendars, clocks, etc.)

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1.2 Patterns in Numbers

Among the most basic patterns that occur in mathematics are patterns of numbers, particularly patterns of whole numbers:

0, 1, 2, 3, 4, ...

The branch of Mathematics that studies patterns in whole numbers is called **number theory**.

Number sequences are the most basic and among the most fascinating types of patterns that mathematicians study.

Table 1 shows some key number sequences that are studied in Mathematics.

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Table 1: Examples of number sequences

1, 1, 1, 1, 1, 1, 1, ...	(All 1's)
1, 2, 3, 4, 5, 6, 7, ...	(Counting numbers)
1, 3, 5, 7, 9, 11, 13, ...	(Odd numbers)
2, 4, 6, 8, 10, 12, 14, ...	(Even numbers)
1, 3, 6, 10, 15, 21, 28, ...	(Triangular numbers)
1, 4, 9, 16, 25, 36, 49, ...	(Squares)
1, 8, 27, 64, 125, 216, ...	(Cubes)
1, 2, 3, 5, 8, 13, 21, ...	(Virahānka numbers)
1, 2, 4, 8, 16, 32, 64, ...	(Powers of 2)
1, 3, 9, 27, 81, 243, 729, ...	(Powers of 3)

Figure it Out

1. Can you recognise the pattern in each of the sequences in Table 1?
2. Rewrite each sequence of Table 1 in your notebook, along with the next three numbers in each sequence! After each sequence, write in your own words what is the rule for forming the numbers in the sequence.

Math Talk

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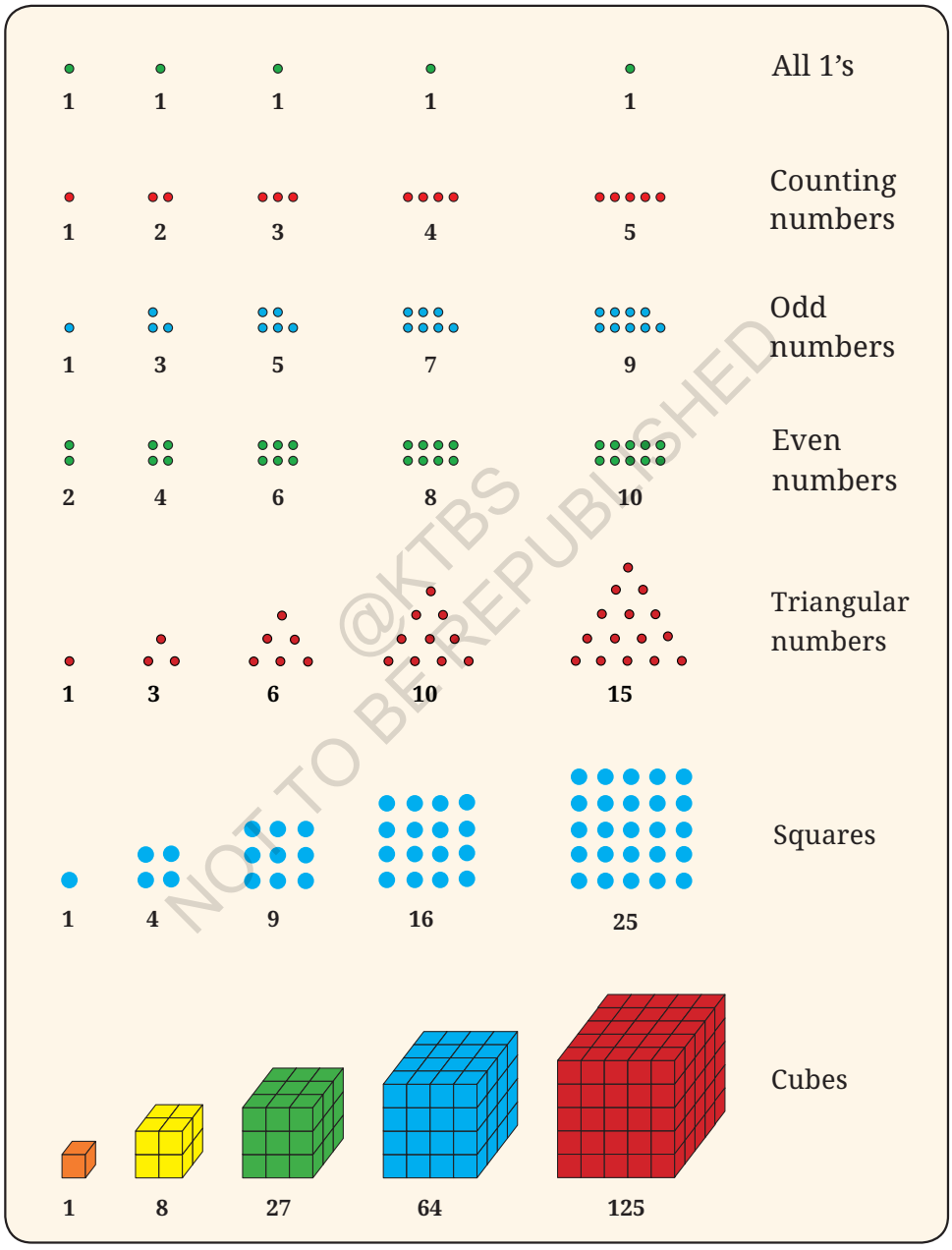
1.3 Visualising Number Sequences

Many number sequences can be visualised using pictures. Visualising mathematical objects through pictures or diagrams can be a very fruitful way to understand mathematical patterns and concepts.

Let us represent the first seven sequences in Table 1 using the following pictures.

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Table 2: Pictorial representation of some number sequences



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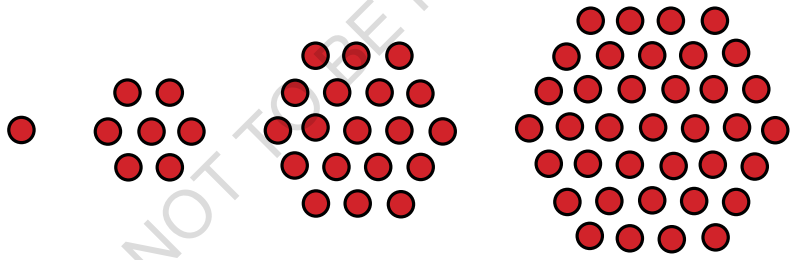
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Figure it Out

Math Talk

1. Copy the pictorial representations of the number sequences in Table 2 in your notebook, and draw the next picture for each sequence!
2. Why are 1, 3, 6, 10, 15, ... called **triangular numbers**? Why are 1, 4, 9, 16, 25, ... called **square numbers** or **squares**? Why are 1, 8, 27, 64, 125, ... called **cubes**?
3. You will have noticed that 36 is both a triangular number and a square number! That is, 36 dots can be arranged perfectly both in a triangle and in a square. Make pictures in your notebook illustrating this!

   This shows that the same number can be represented differently, and play different roles, depending on the context. Try representing some other numbers pictorially in different ways!
4. What would you call the following sequence of numbers?



1 **7** **19** **37**

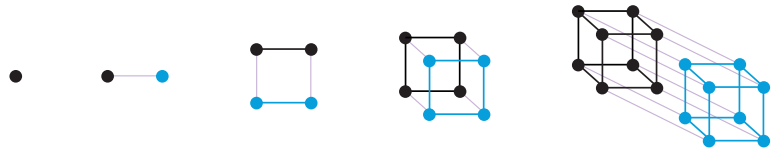
That's right, they are called **hexagonal numbers**! Draw these in your notebook. What is the next number in the sequence?

5. Can you think of pictorial ways to visualise the sequence of Powers of 2? Powers of 3?

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Here is one possible way of thinking about Powers of 2:



1.4 Relations among Number Sequences

Sometimes, number sequences can be related to each other in surprising ways.

Example: What happens when we start adding up odd numbers?

$1 = 1$
$1 + 3 = 4$
$1 + 3 + 5 = 9$
$1 + 3 + 5 + 7 = 16$
$1 + 3 + 5 + 7 + 9 = 25$
$1 + 3 + 5 + 7 + 9 + 11 = 36$
⋮

This is a really beautiful pattern!

Why does this happen? Do you think it will happen forever?

The answer is that the pattern does happen forever. But why? As mentioned earlier, the reason why the pattern happens is just as important and exciting as the pattern itself.

A picture can explain it

Visualising with a picture can help explain the phenomenon. Recall that square numbers are made by counting the number of dots in a square grid.

How can we partition the dots in a square grid into odd numbers of dots: 1, 3, 5, 7,... ?

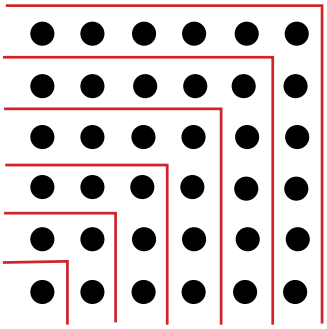
Math Talk

Think about it for a moment before reading further!

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Here is how it can be done:



This picture now makes it evident that

$$1 + 3 + 5 + 7 + 9 + 11 = 36.$$

Because such a picture can be made for a square of any size, this explains why adding up odd numbers gives square numbers.

By drawing a similar picture, can you say what is the sum of the first 10 odd numbers?

Now by imagining a similar picture, or by drawing it partially, as needed, can you say what is the sum of the first 100 odd numbers?

Another example of such a relation between sequences: Adding up and down

Let us look at the following pattern:

$$1 = 1$$
$$1 + 2 + 1 = 4$$
$$1 + 2 + 3 + 2 + 1 = 9$$
$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$$
$$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25$$
$$1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36$$
$$\vdots$$

This seems to be giving yet another way of getting the square numbers—by adding the counting numbers up and then down!

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Can you find a similar pictorial explanation?

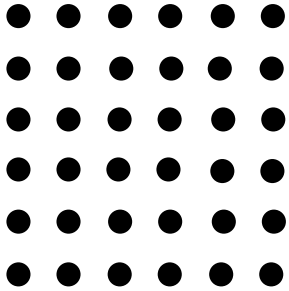


Figure it Out

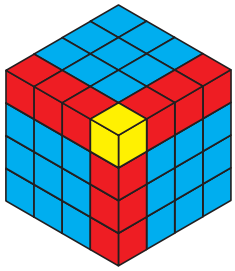


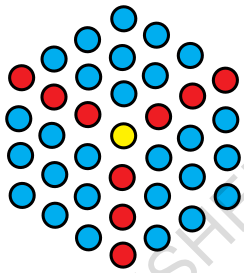
1. Can you find a similar pictorial explanation for why adding counting numbers up and down, i.e., 1, 1 + 2 + 1, 1 + 2 + 3 + 2 + 1, ..., gives square numbers?
2. By imagining a large version of your picture, or drawing it partially, as needed, can you see what will be the value of 1 + 2 + 3 + ··· + 99 + 100 + 99 + ··· + 3 + 2 + 1?
3. Which sequence do you get when you start to add the All 1's sequence up? What sequence do you get when you add the All 1's sequence up and down?
4. Which sequence do you get when you start to add the counting numbers up? Can you give a smaller pictorial explanation?
5. What happens when you add up pairs of consecutive triangular numbers? That is, take 1 + 3, 3 + 6, 6 + 10, 10 + 15, ... Which sequence do you get? Why? Can you explain it with a picture?
6. What happens when you start to add up powers of 2 starting with 1, i.e., take 1, 1 + 2, 1 + 2 + 4, 1 + 2 + 4 + 8, ... ? Now add 1 to each of these numbers—what numbers do you get? Why does this happen?

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7. What happens when you multiply the triangular numbers by 6 and add 1? Which sequence do you get? Can you explain it with a picture?
8. What happens when you start to add up hexagonal numbers, i.e., take 1, 1 + 7, 1 + 7 + 19, 1 + 7 + 19 + 37, ... ? Which sequence do you get? Can you explain it using a picture of a cube?





9. Find your own patterns or relations in and among the sequences in Table 1. Can you explain why they happen with a picture or otherwise?

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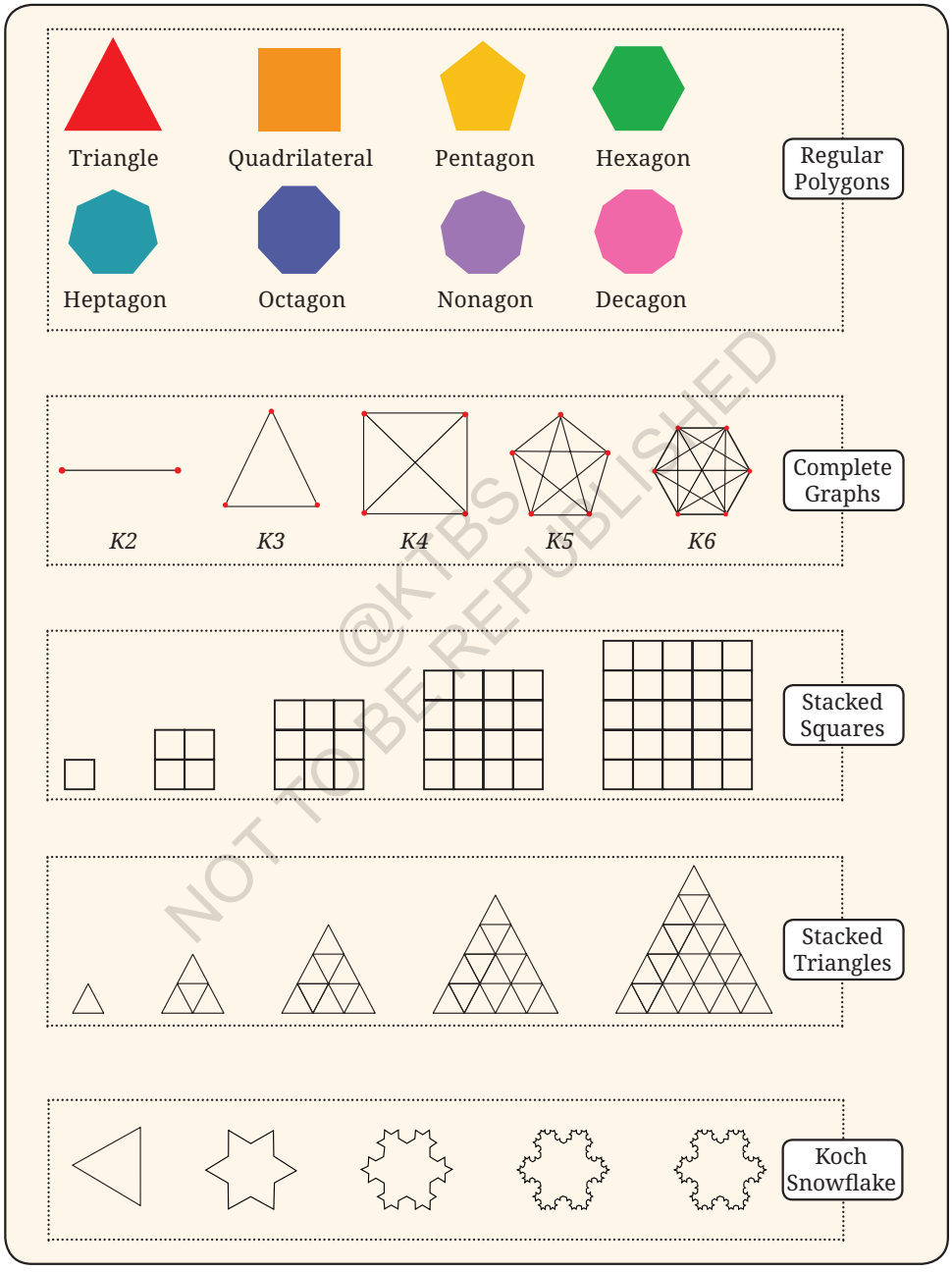
1.5 Patterns in Shapes

Other important and basic patterns that occur in Mathematics are **patterns of shapes.** These shapes may be in one, two, or three dimensions (1D, 2D, or 3D)—or in even more dimensions. The branch of Mathematics that studies patterns in shapes is called **geometry.**

Shape sequences are one important type of shape pattern that mathematicians study. Table 3 shows a few key shape sequences that are studied in Mathematics.

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Table 3: Examples of shape sequences



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Figure it Out

1. Can you recognise the pattern in each of the sequences in Table 3?
2. Try and redraw each sequence in Table 3 in your notebook. Can you draw the next shape in each sequence? Why or why not? After each sequence, describe in your own words what is the rule or pattern for forming the shapes in the sequence.

Math Talk

1.6 Relation to Number Sequences

Often, shape sequences are related to number sequences in surprising ways. Such relationships can be helpful in studying and understanding both the shape sequence and the related number sequence.

Example: The number of sides in the shape sequence of Regular Polygons is given by the counting numbers starting at 3, i.e., 3, 4, 5, 6, 7, 8, 9, 10, ... That is why these shapes are called, respectively, **regular triangle**, **quadrilateral** (i.e., **square**), **pentagon**, **hexagon**, **heptagon**, **octagon**, **nonagon**, **decagon**, etc., respectively.

The word 'regular' refers to the fact that these shapes have equal-length sides and also equal 'angles' (i.e., the sides look the same and the corners also look the same). We will discuss angles in more depth in the next chapter.

The other shape sequences in Table 3 also have beautiful relationships with number sequences.

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Figure it Out

1. Count the number of sides in each shape in the sequence of Regular Polygons. Which number sequence do you get? What about the number of corners in each shape in the sequence of Regular Polygons? Do you get the same number sequence? Can you explain why this happens?
2. Count the number of lines in each shape in the sequence of Complete Graphs. Which number sequence do you get? Can you explain why?

Try This

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3. How many little squares are there in each shape of the sequence of Stacked Squares? Which number sequence does this give? Can you explain why?
4. How many little triangles are there in each shape of the sequence of Stacked Triangles? Which number sequence does this give? Can you explain why? (*Hint*: In each shape in the sequence, how many triangles are there in each row?)
5. To get from one shape to the next shape in the Koch Snowflake sequence, one replaces each line segment '—' by a 'speed bump' —/\—. As one does this more and more times, the changes become tinier and tinier with very very small line segments. How many total line segments are there in each shape of the Koch Snowflake? What is the corresponding number sequence? (The answer is 3, 12, 48, ..., i.e., 3 times Powers of 4; this sequence is not shown in Table 1.)



SUMMARY

- Mathematics may be viewed as the search for patterns and for the explanations as to why those patterns exist.
- Among the most basic patterns that occur in mathematics are **number sequences.**
- Some important examples of number sequences include the counting numbers, odd numbers, even numbers, square numbers, triangular numbers, cube numbers, Virahānka numbers, and powers of 2.
- Sometimes number sequences can be related to each other in beautiful and remarkable ways. For example, adding up the sequence of odd numbers starting with 1 gives square numbers.
- Visualising number sequences using pictures can help to understand sequences and the relationships between them.
- **Shape sequences** are another basic type of pattern in mathematics. Some important examples of shape sequences include regular polygons, complete graphs, stacked triangles and squares, and Koch snowflake iterations. Shape sequences also exhibit many interesting relationships with number sequences.

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2 LINES AND ANGLES

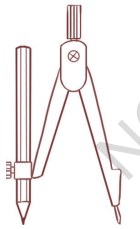
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In this chapter, we will explore some of the most basic ideas of geometry including points, lines, rays, line segments and angles. These ideas form the building blocks of 'plane geometry', and will help us in understanding more advanced topics in geometry such as the construction and analysis of different shapes.

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2.1 Point

Mark a dot on the paper with a sharp tip of a pencil. The sharper the tip, the thinner will be the dot. This tiny dot will give you an idea of a point. A point determines a precise location, but it has no length, breadth or height. Some models for a point are given below.



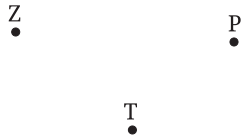
The tip of a compass



The sharpened end of a pencil

The pointed end of a needle

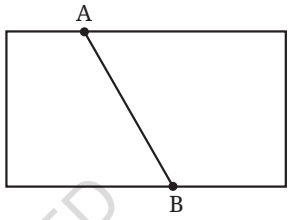
If you mark three points on a piece of paper, you may be required to distinguish these three points. For this purpose, each of the three points may be denoted by a single capital letter such as



Z, P and T. These points are read as 'Point Z', 'Point P' and 'Point T'. Of course, the dots represent precise locations and must be imagined to be invisibly thin.

2.2 Line Segment

Fold a piece of paper and unfold it. Do you see a crease? This gives the idea of a line segment. It has two end points, A and B.



Mark any two points A and B on a sheet of paper. Try to connect A to B by various routes (Fig. 2.1).

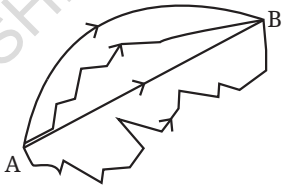


Fig. 2.1

What is the shortest route from A to B? This shortest path from Point A to Point B (including A and B) as shown here is called the **line segment** from A to B. It is denoted by either $\overline{AB}$ or $\overline{BA}$. The points A and B are called the **end points** of the line segment $\overline{AB}$.

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2.3 Line

Imagine that the line segment from A to B (i.e., $\overline{AB}$) is extended beyond A in one direction and beyond B in the other direction without any end (see Fig. 2.2). This is a model for a **line.** Do you think you can draw a complete picture of a line? No. Why?

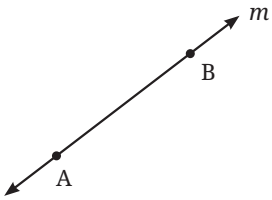


Fig. 2.2

A line through two points A and B is written as $\overleftrightarrow{AB}$. It extends forever in both directions. Sometimes a line is denoted by a letter like *l* or *m*.

Observe that any two points determine a unique line that passes through both of them.

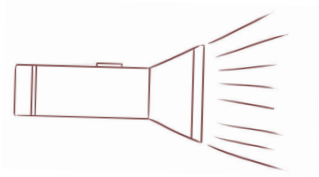
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2.4 Ray

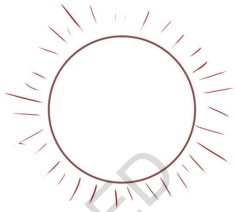
A **ray** is a portion of a line that starts at one point (called the **starting point** or **initial point** of the ray) and goes on endlessly in a direction. The following are some models for a ray:



Beam of light from a lighthouse



Ray of light from a torch



Sun rays

Look at the diagram (Fig. 2.3) of a ray. Two points are marked on it. One is the starting point A and the other is a point P on the path of the ray. We then denote the ray by $\overrightarrow{AP}$.

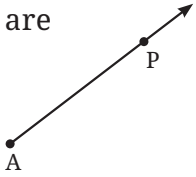


Fig. 2.3

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Figure it Out

1. Rihan marked a point on a piece of paper. How many lines can he draw that pass through the point?

   Sheetal marked two points on a piece of paper. How many different lines can she draw that pass through both of the points?

   Can you help Rihan and Sheetal find their answers?

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2. Name the line segments in Fig. 2.4. Which of the five marked points are on exactly one of the line segments? Which are on two of the line segments?

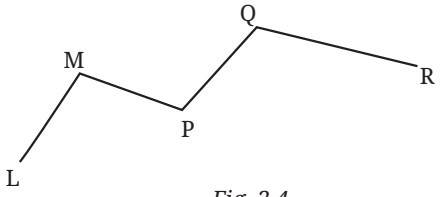


Fig. 2.4

3. Name the rays shown in Fig. 2.5. Is T the starting point of each of these rays?

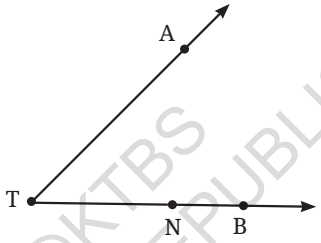


Fig. 2.5

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4. Draw a rough figure and write labels appropriately to illustrate each of the following:
   a. $\overleftrightarrow{OP}$ and $\overleftrightarrow{OQ}$ meet at O.
   b. $\overrightarrow{XY}$ and $\overleftrightarrow{PQ}$ intersect at point M.
   c. Line l contains points E and F but not point D.
   d. Point P lies on AB.

5. In Fig. 2.6, name:
   a. Five points
   b. A line
   c. Four rays
   d. Five line segments

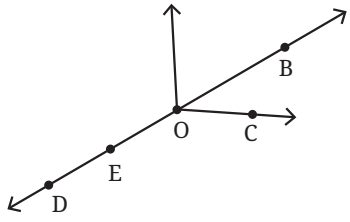


Fig. 2.6

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6. Here is a ray $\overrightarrow{OA}$ (Fig. 2.7). It starts at O and passes through the point A. It also passes through the point B.
   a. Can you also name it as $\overrightarrow{OB}$? Why?
   b. Can we write $\overrightarrow{OA}$ as $\overrightarrow{AO}$? Why or why not?

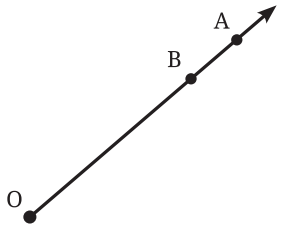


Fig. 2.7

2.5 Angle

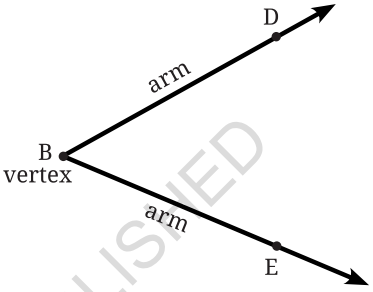


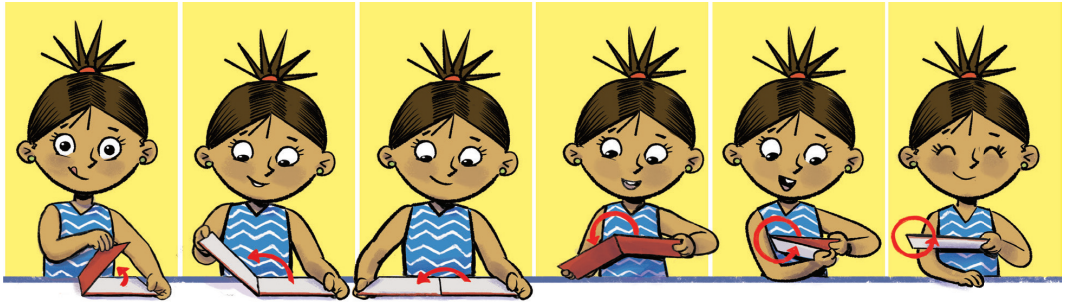
Fig. 2.8

An angle is formed by two rays having a common starting point. Here is an angle formed by rays $\overrightarrow{BD}$ and $\overrightarrow{BE}$ where B is the common starting point (Fig. 2.8).

The point B is called the **vertex** of the angle, and the rays $\overrightarrow{BD}$ and $\overrightarrow{BE}$ are called the **arms** of the angle. How can we name this angle? We can simply use the vertex and say that it is the Angle B. To be clearer, we use a point on each of the arms together with the vertex to name the angle. In this case, we name the angle as Angle DBE or Angle EBD. The word angle can be replaced by the symbol '$\angle$', i.e., $\angle$DBE or $\angle$EBD. Note that in specifying the angle, the vertex is always written as the middle letter.

To indicate an angle, we use a small curve at the vertex (refer to Fig. 2.9).

Vidya has just opened her book. Let us observe her opening the cover of the book in different scenarios.



Case 1 Case 2 Case 3 Case 4 Case 5 Case 6

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Do you see angles being made in each of these cases? Can you mark their arms and vertex?

Which angle is greater—the angle in Case 1 or the angle in Case 2?

Just as we talk about the size of a line based on its length, we also talk about the size of an angle based on its amount of rotation.

So, the angle in Case 2 is greater as in this case she needs to rotate the cover more. Similarly, the angle in Case 3 is even larger than that of Case 2, because there is even more rotation, and Cases 4, 5, and 6 are successively larger angles with more and more rotation.

The size of an angle is the amount of rotation or turn that is needed about the vertex to move the first ray to the second ray.

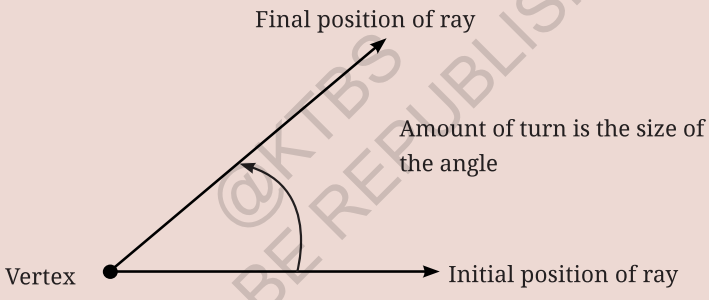


Fig. 2.9

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Let's look at some other examples where angles arise in real life by rotation or turn:

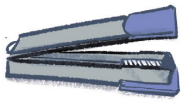
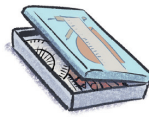
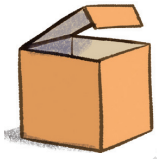
- In a compass or divider, we turn the arms to form an angle. The vertex is the point where the two arms are joined. Identify the arms and vertex of the angle.
- A pair of scissors has two blades. When we open them (or 'turn them') to cut something, the blades form an angle. Identify the arms and the vertex of the angle.



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- Look at the pictures of spectacles, wallet and other common objects. Identify the angles in them by marking out their arms and vertices.



Do you see how these angles are formed by turning one arm with respect to the other?

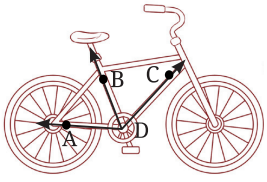
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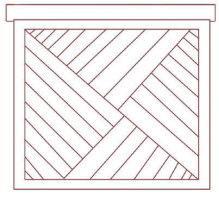
Teacher's Note

Teacher needs to organise various activities with the students to recognise the size of an angle as a measure of rotation.

Figure it Out

1. Can you find the angles in the given pictures? Draw the rays forming any one of the angles and name the vertex of the angle.





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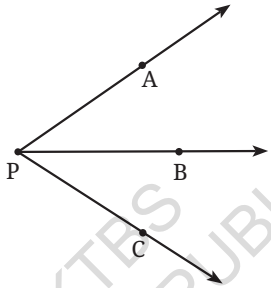




2. Draw and label an angle with arms *ST* and *SR*.

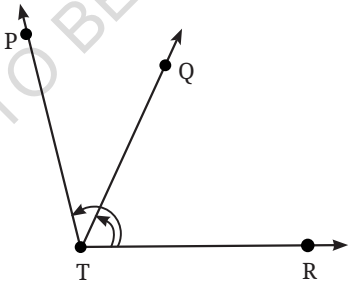
3. Explain why $\angle APC$ cannot be labelled as $\angle P$.

Math Talk



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4. Name the angles marked in the given figure.



5. Mark any three points on your paper that are not on one line. Label them A, B, C. Draw all possible lines going through pairs of these points. How many lines do you get? Name them. How many angles can you name using A, B, C? Write them down, and mark each of them with a curve as in Fig. 2.9.

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6. Now mark any four points on your paper so that no three of them are on one line. Label them A, B, C, D. Draw all possible lines going through pairs of these points. How many lines do you get? Name them. How many angles can you name using A, B, C, D? Write them all down, and mark each of them with a curve as in Fig. 2.9.

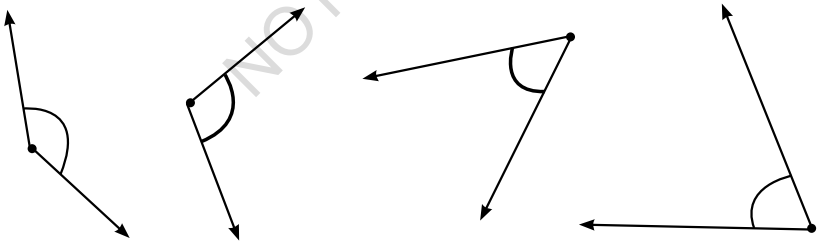
2.6 Comparing Angles

Look at these animals opening their mouths. Do you see any angles here? If yes, mark the arms and vertex of each one. Some mouths are open wider than others; the more the turning of the jaws, the larger the angle! Can you arrange the angles in this picture from smallest to largest?



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Is it always easy to compare two angles?



Math Talk

Here are some angles. Label each of the angles. How will you compare them?

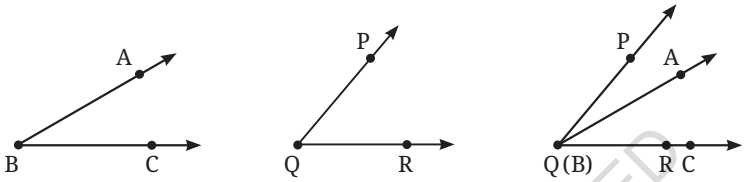
Draw a few more angles; label them and compare.

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Comparing angles by superimposition

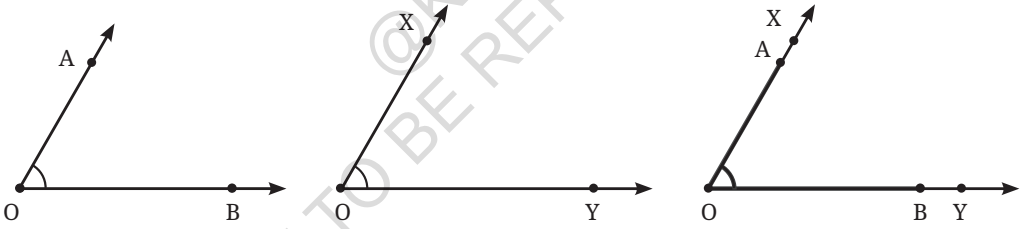
Any two angles can be compared by placing them one over the other, i.e., by superimposition. While superimposing, the vertices of the angles must overlap.

After superimposition, it becomes clear which angle is smaller and which is larger.



The picture shows the two angles superimposed. It is now clear that ∠PQR is larger than ∠ABC.

Equal angles. Now consider ∠AOB and ∠XOY in the figure. Which is greater?



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The corners of both of these angles match and the arms overlap with each other, i.e., OA ↔ OX and OB ↔ OY. So, the angles are **equal** in size.

The reason these angles are considered to be equal in size is because when we visualise each of these angles as being formed out of rotation, we can see that there is an equal amount of rotation needed to move $\overrightarrow{OB}$ to $\overrightarrow{OA}$ and $\overrightarrow{OY}$ to $\overrightarrow{OX}$.

From the point of view of superimposition, when two angles are superimposed, and the common vertex and the two rays of both angles lie on top of each other, then the sizes of the angles are equal.

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Where else do we use superimposition to compare?

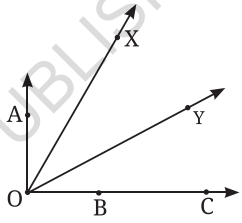
Math Talk

Figure it Out

1. Fold a rectangular sheet of paper, then draw a line along the fold created. Name and compare the angles formed between the fold and the sides of the paper. Make different angles by folding a rectangular sheet of paper and compare the angles. Which is the largest and smallest angle you made?

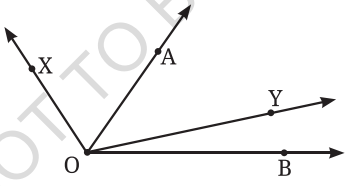


2. In each case, determine which angle is greater and why.
   a. ∠AOB or ∠XOY
   b. ∠AOB or ∠XOB
   c. ∠XOB or ∠XOC
   Discuss with your friends on how you decided which one is greater.



Math Talk

3. Which angle is greater: ∠XOY or ∠AOB? Give reasons.



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Comparing angles without superimposition

Two cranes are arguing about who can open their mouth wider, i.e., who is making a bigger angle.

Let us first draw their angles. How do we know which one is bigger? As seen

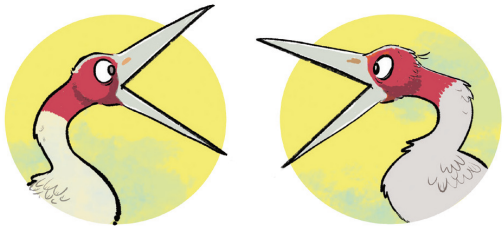


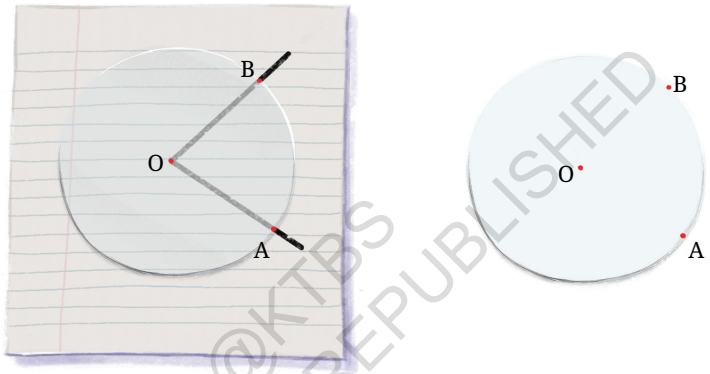
Fig. 2.10

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before, one could trace these angles, superimpose them and then check. But can we do it without superimposition?

Suppose we have a transparent circle which can be moved and placed on figures. Can we use this for comparison?

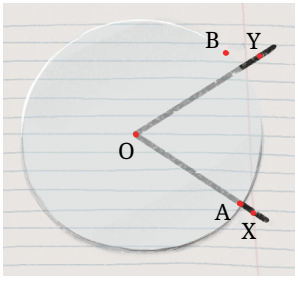
Let us place the circular paper on the angle made by the first crane. The circle is placed in such a way that its centre is on the vertex of the angle. Let us mark the points A and B on the edge circle at the points where the arms of the angle pass through the circle.



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Can we use this to find out if this angle is greater than, or equal to or smaller than the angle made by the second crane?

Let us place it on the angle made by the second crane so that the vertex coincides with the centre of the circle and one of the arms passes through OA.



Can you now tell which angle is bigger?

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Which crane was making the bigger angle?
If you can make a circular piece of transparent paper, try this method to compare the angles in Fig. 2.10 with each other.

Teacher's Note

A teacher needs to check the understanding of the students around the notion of an angle. Sometimes students might think that increasing the length of the arms of the angle increases the angle. For this, various situations should be posed to the students to check their understanding on the same.

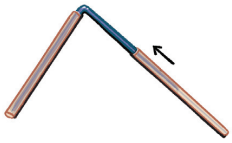
2.7 Making Rotating Arms

Let us make 'rotating arms' using two paper straws and a paper clip by following these steps:

1. Take two paper straws and a paper clip.



2. Insert the straws into the arms of the paper clip.



3. Your rotating arm is ready!



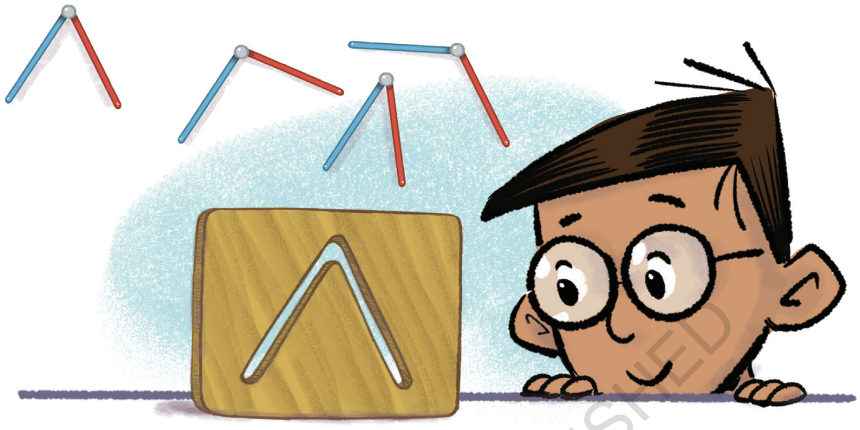
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Make several 'rotating arms' with different angles between the arms. Arrange the angles you have made from smallest to largest by comparing and using superimposition.

Passing through a slit: Collect a number of rotating arms with different angles; do not rotate any of the rotating arms during this activity.

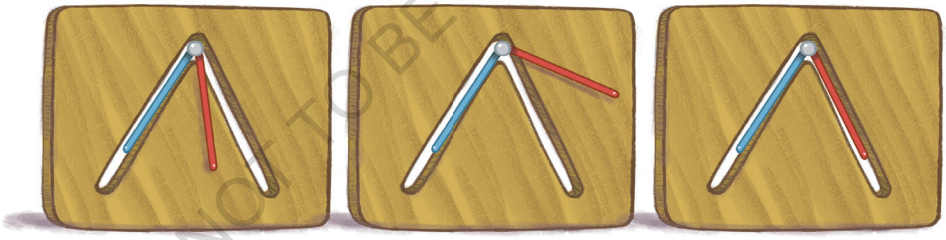
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Take a cardboard and make an angle-shaped slit as shown below by tracing and cutting out the shape of one of the rotating arms.



Now, shuffle and mix up all the rotating arms. Can you identify which of the rotating arms will pass through the slit?

The correct one can be found by placing each of the rotating arms over the slit. Let us do this for some of the rotating arms:



Slit angle is greater than the arms' angle. The arms will not go through the slit.

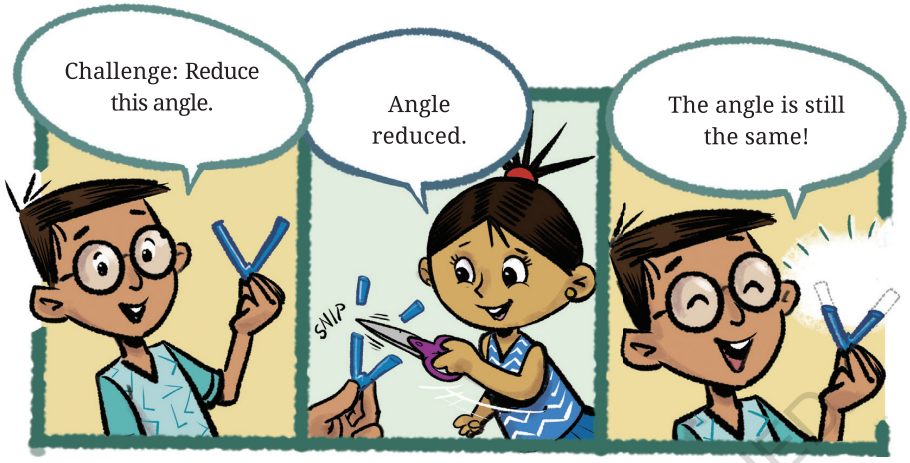
Slit angle is less than the arms' angle. The arms will not go through the slit.

Slit angle is equal to the arms' angle. The arms will go through the slit.

Only the pair of rotating arms where the angle is equal to that of the slit passes through the slit. Note that the possibility of passing through the slit depends only on the angle between the rotating arms and not on their lengths (as long as they are shorter than the length of the slit).

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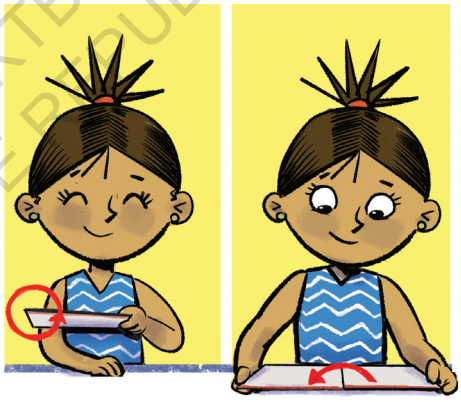
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2.8 Special Types of Angles

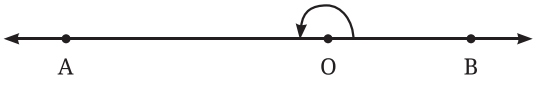
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Let us go back to Vidya's notebook and observe her opening the cover of the book in different scenarios.



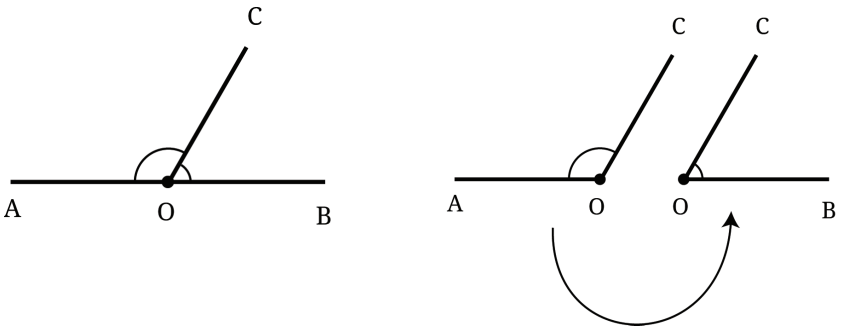
She makes a full turn of the cover when she has to write while holding the book in her hand.

She makes a half turn of the cover when she has to open it on her table. In this case, observe the arms of the angle formed. They lie in a straight line. Such an angle is called a **straight angle**.



Let us consider a straight angle ∠AOB. Observe that any ray $\overrightarrow{OC}$ divides it into two angles, ∠AOC and ∠COB.

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Is it possible to draw $\overrightarrow{OC}$ such that the two angles are equal to each other in size?

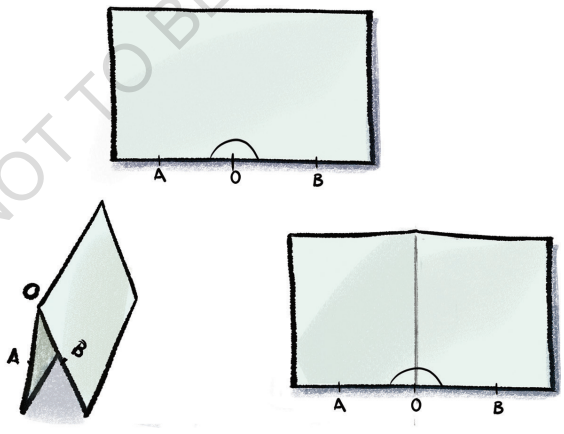
Math Talk

Let's Explore

We can try to solve this problem using a piece of paper. Recall that when a fold is made, it creates a crease which is straight.

Take a rectangular piece of paper and on one of its sides, mark the straight angle AOB. By folding, try to get a line (crease) passing through O that divides ∠AOB into two equal angles.

How can it be done?



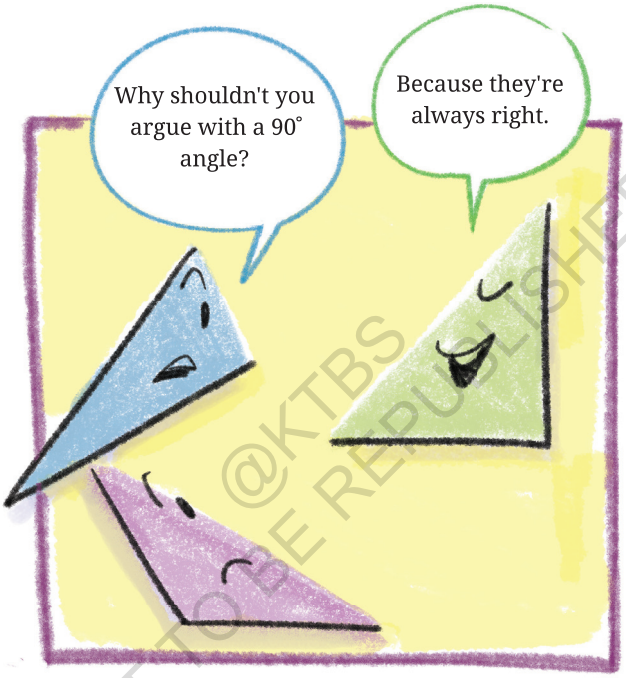
Fold the paper such that OB overlaps with OA. Observe the crease and the two angles formed.

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Justify why the two angles are equal. Is there a way to superimpose and check? Can this superimposition be done by folding?

Each of these equal angles formed are called right angles. So, a straight angle contains two **right angles**.



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If a straight angle is formed by half of a full turn, how much of a full turn will form a right angle?

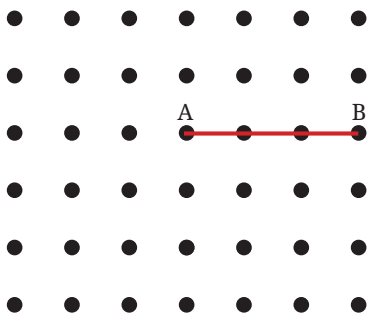
Observe that a right angle resembles the shape of an 'L'. An angle is a right angle only if it is exactly half of a straight angle. Two lines that meet at right angles are called **perpendicular** lines.

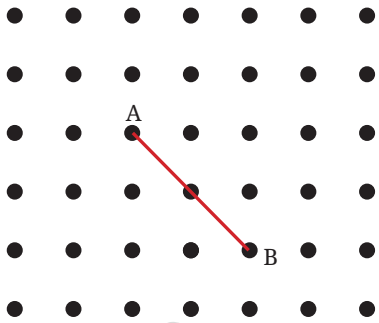
Figure it Out

1. How many right angles do the windows of your classroom contain? Do you see other right angles in your classroom?

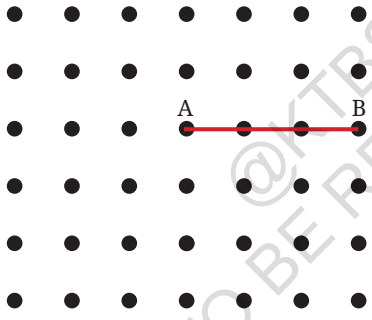
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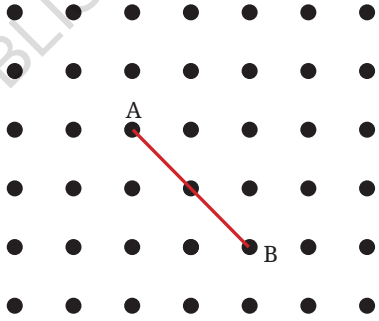
2. Join A to other grid points in the figure by a straight line to get a straight angle. What are all the different ways of doing it?





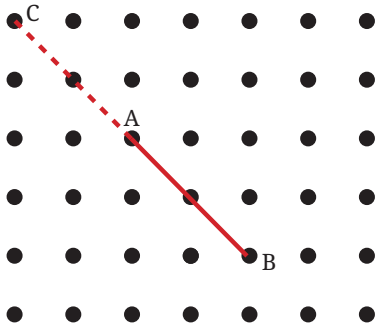
3. Now join A to other grid points in the figure by a straight line to get a right angle. What are all the different ways of doing it?





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Hint: Extend the line further as shown in the figure below. To get a right angle at A, we need to draw a line through it that divides the straight angle CAB into two equal parts.

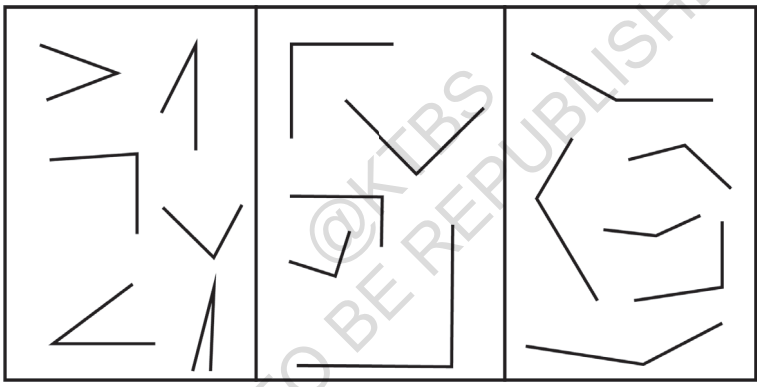


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4. Get a slanting crease on the paper. Now, try to get another crease that is perpendicular to the slanting crease.
   a. How many right angles do you have now? Justify why the angles are exact right angles.
   b. Describe how you folded the paper so that any other person who doesn't know the process can simply follow your description to get the right angle.

Classifying Angles

Angles are classified in three groups as shown below. Right angles are shown in the second group. What could be the common feature of the other two groups?



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In the first group, all angles are less than a right angle or in other words, less than a quarter turn. Such angles are called **acute angles**.

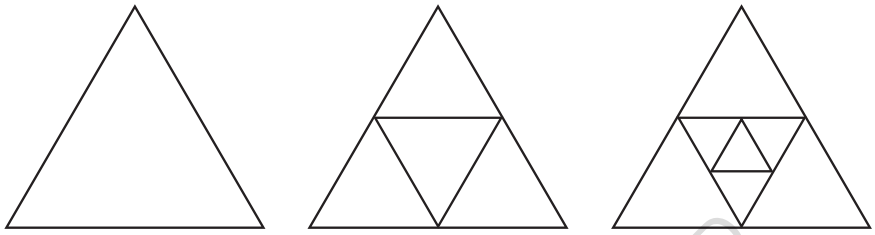
In the third group, all angles are greater than a right angle but less than a straight angle. The turning is more than a quarter turn and less than a half turn. Such angles are called **obtuse angles**.

Figure it Out

1. Identify acute, right, obtuse and straight angles in the previous figures.
2. Make a few acute angles and a few obtuse angles. Draw them in different orientations.

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3. Do you know what the words acute and obtuse mean? Acute means sharp and obtuse means blunt. Why do you think these words have been chosen?
4. Find out the number of acute angles in each of the figures below.



What will be the next figure and how many acute angles will it have? Do you notice any pattern in the numbers?

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2.9 Measuring Angles

We have seen how to compare two angles. But can we actually quantify how big an angle is using a number without having to compare it to another angle?

We saw how various angles can be compared using a circle. Perhaps a circle could be used to assign measures for angles?

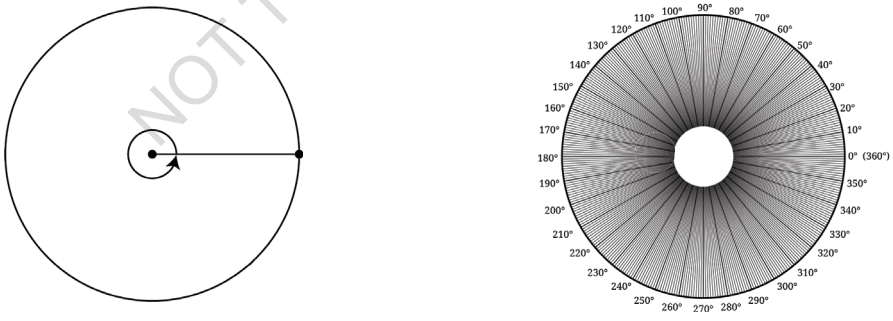


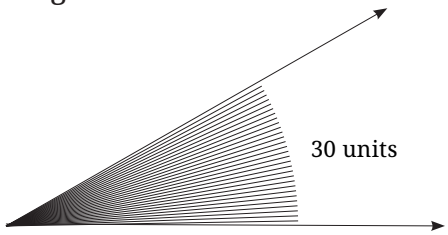
Fig. 2.12

To assign precise measures to angles, mathematicians came up with an idea. They divided the angle in the centre of the circle into

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360 equal angles or parts. The angle measure of each of these unit parts is 1 degree, which is written as 1°.

This unit part is used to assign measure to any angle: the measure of an angle is the number of 1° unit parts it contains inside it. For example, see this figure:

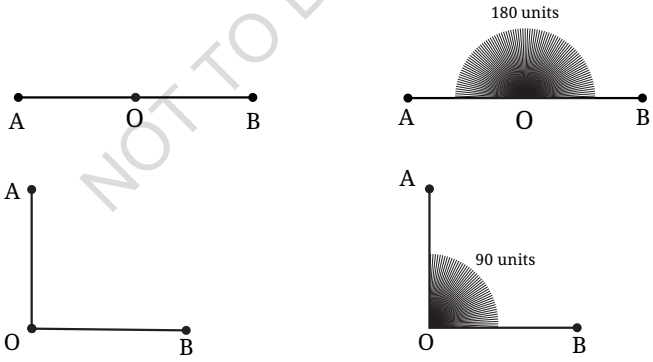


It contains 30 units of 1° angle and so we say that its angle measure is 30°.

Measures of different angles: What is the measure of a full turn in degrees? As we have taken it to contain 360 degrees, its measure is 360°.

What is the measure of a straight angle in degrees? A straight angle is half of a full turn. As a full-turn is 360°, a half turn is 180°. What is the measure of a right angle in degrees? Two right angles together form a straight angle. As a straight angle measures 180°, a right angle measures 90°.

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A pinch of history

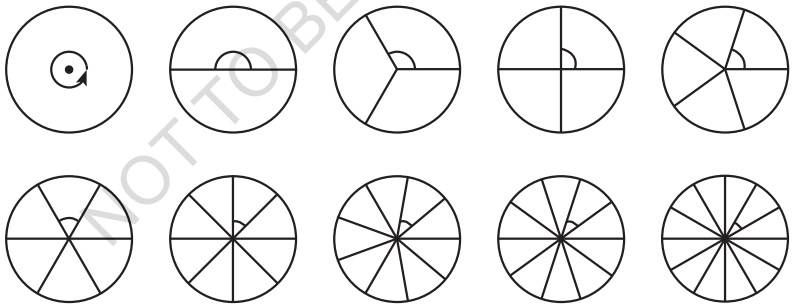
A full turn has been divided into 360°. Why 360? The reason why we use 360° today is not fully known. The division of a circle into

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360 parts goes back to ancient times. The *Rigveda*, one of the very oldest texts of humanity going back thousands of years, speaks of a wheel with 360 spokes (Verse 1.164.48). Many ancient calendars, also going back over 3000 years—such as calendars of India, Persia, Babylonia and Egypt—were based on having 360 days in a year. In addition, Babylonian mathematicians frequently used divisions of 60 and 360 due to their use of sexagesimal numbers and counting by 60s.

Perhaps the most important and practical answer for why mathematicians over the years have liked and continued to use 360 degrees is that 360 is the smallest number that can be evenly divided by all numbers up to 10, aside from 7. Thus, one can break up the circle into 1, 2, 3, 4, 5, 6, 8, 9 or 10 equal parts, and still have a whole number of degrees in each part! Note that 360 is also evenly divisible by 12, the number of months in a year, and by 24, the number of hours in a day. These facts all make the number 360 very useful.

The circle has been divided into 1, 2, 3, 4, 5, 6, 8, 9 10 and 12 parts below. What are the degree measures of the resulting angles? Write the degree measures down near the indicated angles.



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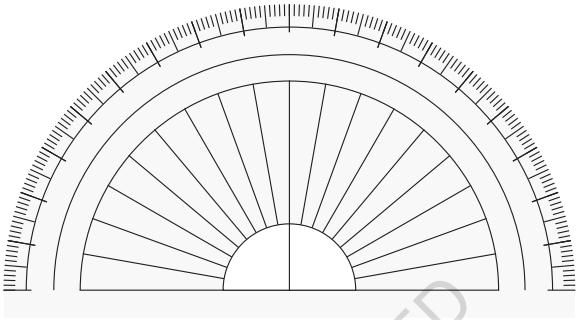
Degree measures of different angles

How can we measure other angles in degrees? It is for this purpose that we have a tool called a **protractor** that is either a circle divided into 360 equal parts as shown in Fig. 2.12 (on page 32), or a half circle divided into 180 equal parts.

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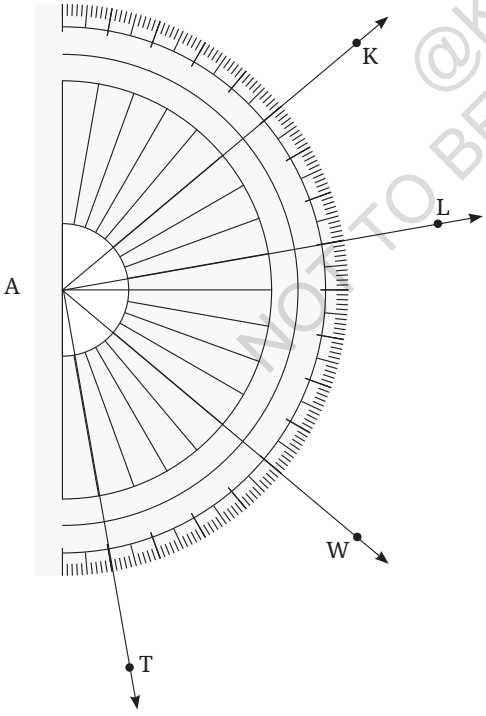
Unlabelled protractor

Here is a protractor. Do you see the straight angle at the center divided into 180 units of 1 degree? Only part of the lines dividing the straight angle are visible, though!



Starting from the marking on the rightmost point of the base, there is a long mark for every 10°. From every such long mark, there is a medium sized mark after 5°.

Figure it out



1. Write the measures of the following angles:

a. ∠ KAL

Notice that the vertex of this angle coincides with the centre of the protractor. So the number of units of 1 degree angle between KA and AL gives the measure of ∠KAL. By counting, we get—

∠KAL = 30°

Making use of the medium sized and large sized marks, is it possible to count the number of units in 5s or 10s?

b. ∠WAL

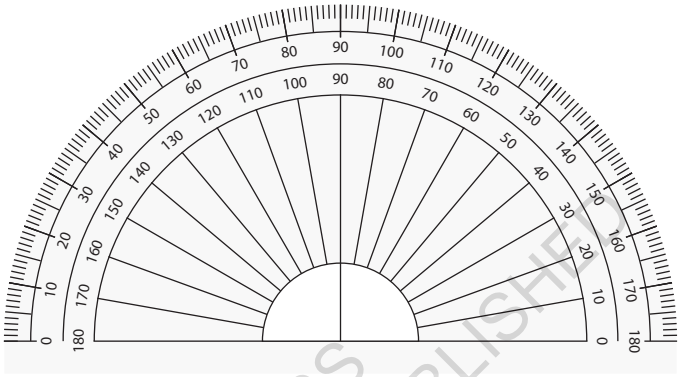
c. ∠TAK

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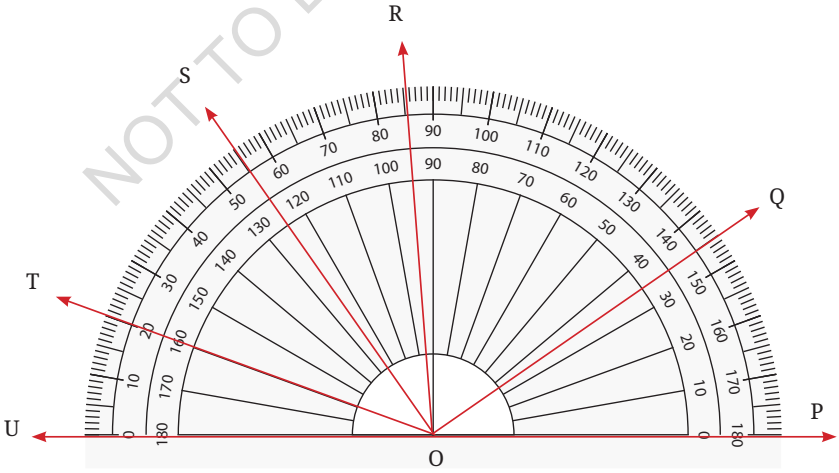
Labelled protractor

This is a protractor that you find in your geometry box. It would appear similar to the protractor above except that there are numbers written on it. Will these make it easier to read the angles?



There are two sets of numbers on the protractor: one increasing from right to left and the other increasing from left to right. Why does it include two sets of numbers?

Name the different angles in the figure and write their measures.



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Did you include angles such as ∠TOQ?

Which set of markings did you use—inner or outer?

What is the measure of ∠TOS?

Can you use the numbers marked to find the angle without counting the number of markings?

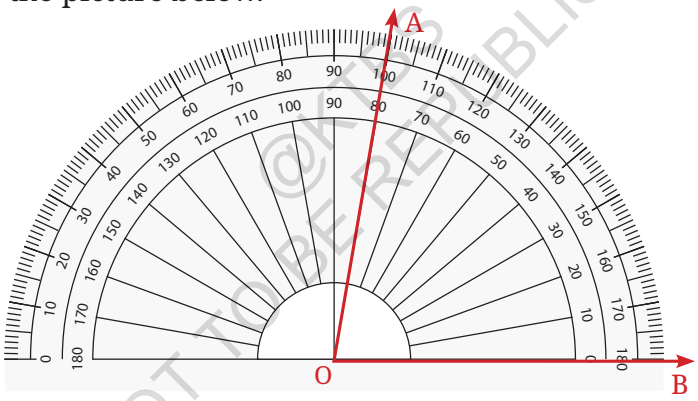
Here, OT and OS pass through the numbers 20 and 55 on the outer scale. How many units of 1 degree are contained between these two arms?

Can subtraction be used here?

How can we measure angles directly without having to subtract?

Place the protractor so the center is on the vertex of the angle.

Align the protractor so that one the arms passes through the 0º mark as in the picture below.



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What is the degree measure of ∠AOB?

Make your own Protractor!

You may have wondered how the different equally spaced markings are made on a protractor. We will now see how we can make some of them!

1. Draw a circle of a convenient radius on a sheet of paper. Cut out the circle (Fig. 2.13). A circle or one full turn is 360°.
2. Fold the circle to get two equal halves and cut it through the crease to get a semicircle. Write '0°' in the bottom right corner of the semicircle.

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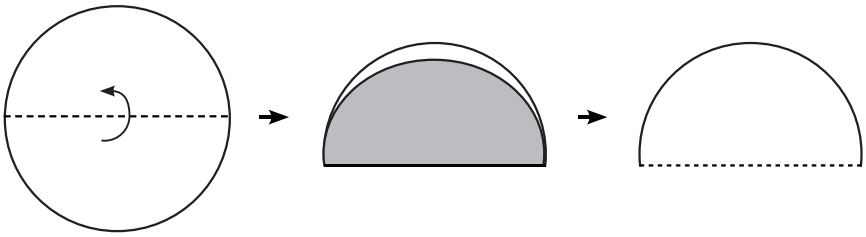


Fig. 2.13

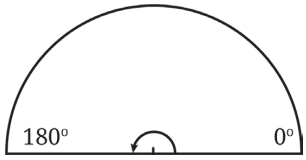
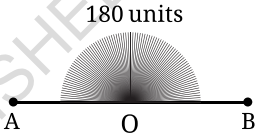


Fig. 2.14

The measure of half a circle is $\frac{1}{2}$ of a full turn. (Fig. 2.14)
So, the measure of half a turn = $\frac{1}{2}$ of ____ = 180°.
Thus, write 180° in the left bottom corner of the semicircle.



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3. Fold the semi-circular sheet in half as shown in Fig. 2.15 to form a quarter circle.

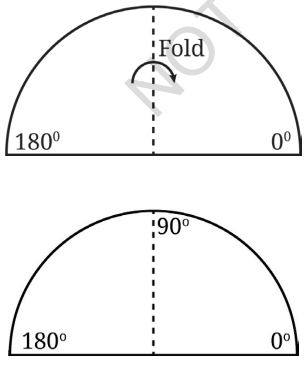
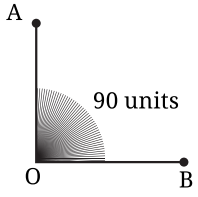


Fig. 2.15

The measure of a quarter circle is $\frac{1}{4}$ of a full turn.
The measure of a $\frac{1}{4}$ turn = $\frac{1}{4}$ of 360° = _____.
Or, the measure of a $\frac{1}{4}$ turn = $\frac{1}{2}$ of a half turn = $\frac{1}{2}$ of 180° = _____.
Thus, mark 90° at the top of the semicircle.



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4. Fold the sheet again as shown in Figs. 2.16 and 2.17:

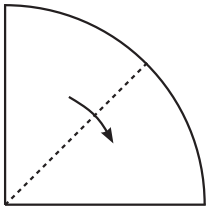


Fig. 2.16

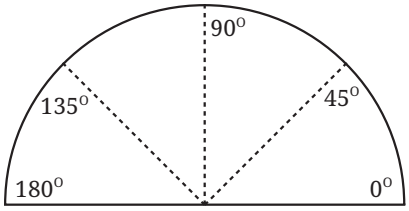


Fig. 2.17

When folded, this is $\frac{1}{8}$ of the circle, or $\frac{1}{8}$ of a turn, or $\frac{1}{8}$ of 360°, or $\frac{1}{4}$ of 180° or $\frac{1}{2}$ of 90° = _____.

The new creases formed give us measures of 45° and 180°– 45° = 135° as shown. Write 45° and 135° at the correct places on the new creases along the edge of the semicircle.

5. Continuing with another half fold as shown in Fig. 2.18, we get an angle of measure _____.

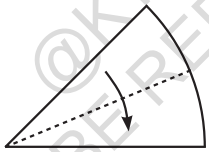


Fig. 2.18

6. Unfold and mark the creases as OB, OC, ..., etc., as shown in Fig. 2.19 and Fig. 2.20.

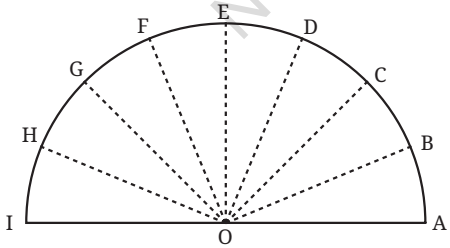


Fig. 2.19

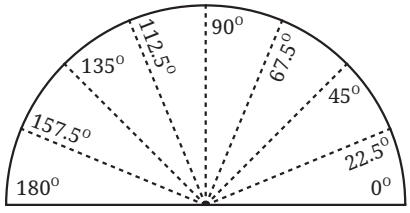


Fig. 2.20

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Think!

In Fig. 2.20, we have ∠AOB = ∠BOC = ∠COD = ∠DOE = ∠EOF = ∠FOG = ∠GOH = ∠HOI=_____. Why?

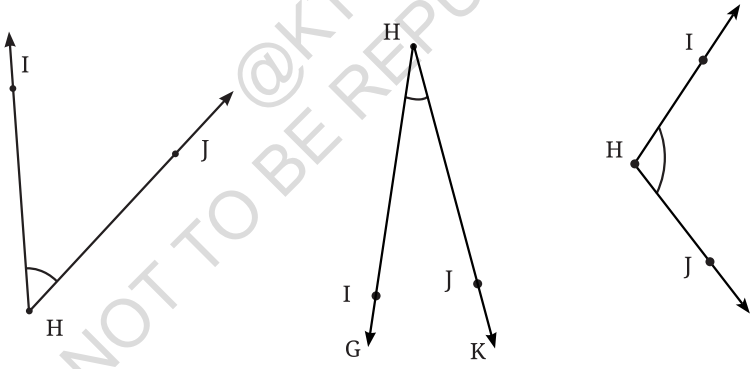
Angle Bisector

At each step, we folded in halves. This process of getting half of a given angle is called **bisecting the angle**. The line that bisects a given angle is called the **angle bisector** of the angle.

Identify the angle bisectors in your handmade protractor. Try to make different angles using the concept of angle bisector through paper folding.

Figure it Out

1. Find the degree measures of the following angles using your protractor.



2. Find the degree measures of different angles in your classroom using your protractor.

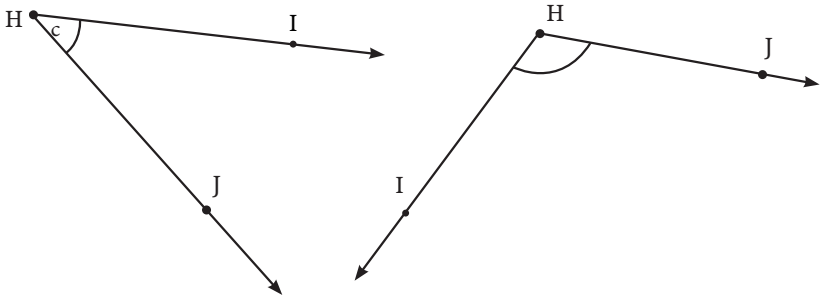
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Teacher's Note

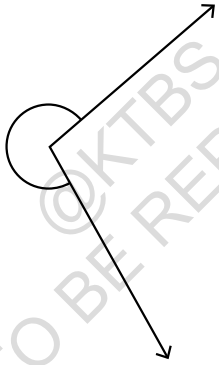
It is important that students make their own protractor and use it to measure different angles before using the standard protractor so that they know the concept behind the marking of the standard protractor.

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3. Find the degree measures for the angles given below. Check if your paper protractor can be used here!



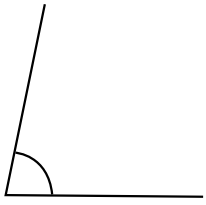
4. How can you find the degree measure of the angle given below using a protractor?



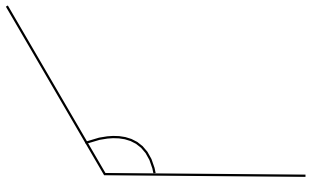
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5. Measure and write the degree measures for each of the following angles:

a.

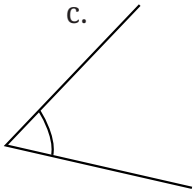


b.

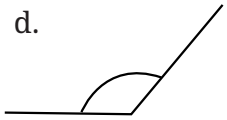


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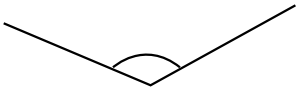
c.



d.



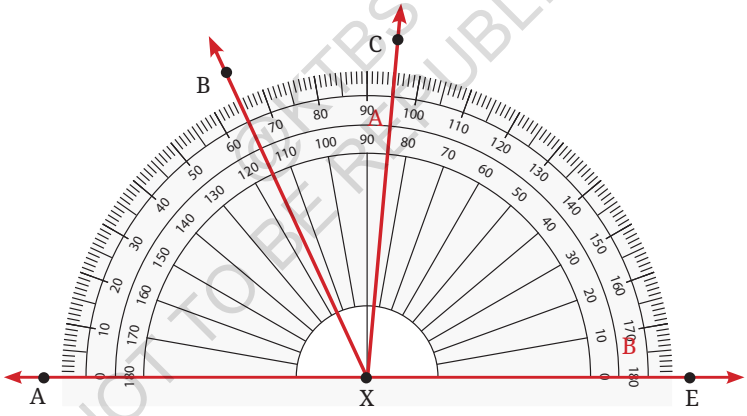
e.



f.

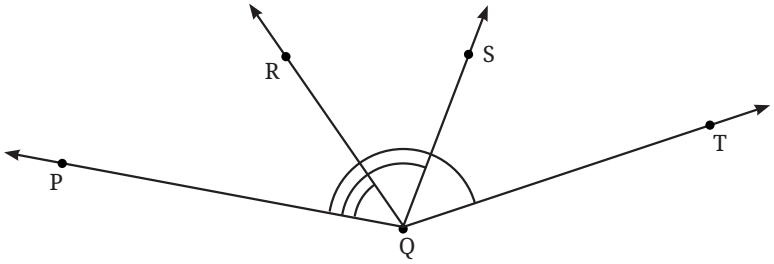


6. Find the degree measures of ∠BXE, ∠CXE, ∠AXB and ∠BXC.



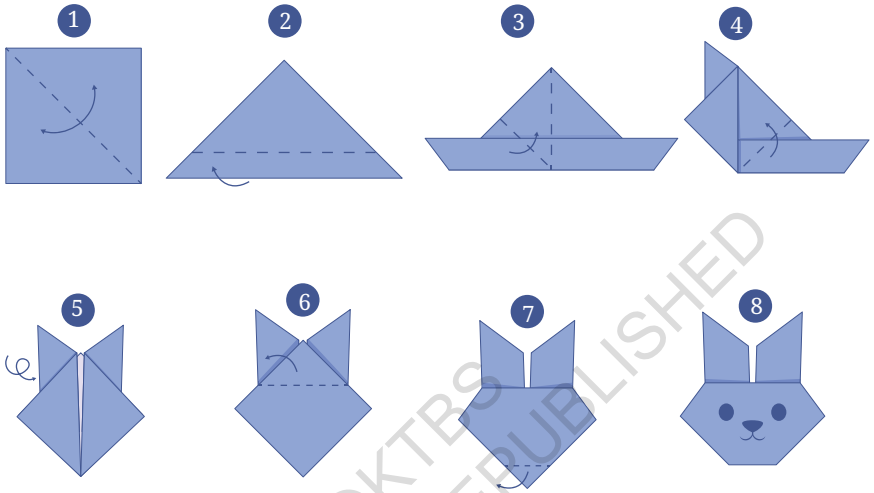
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7. Find the degree measures of ∠PQR, ∠PQS and ∠PQT.



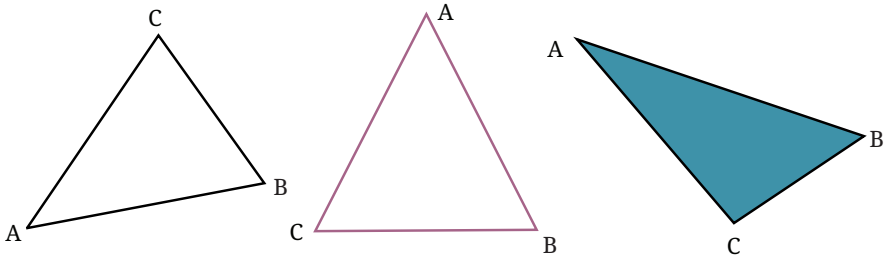
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8. Make the paper craft as per the given instructions. Then, unfold and open the paper fully. Draw lines on the creases made and measure the angles formed.



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9. Measure all three angles of the triangle shown in Fig. 2.21 (a), and write the measures down near the respective angles. Now add up the three measures. What do you get? Do the same for the triangles in Fig. 2.21 (b) and (c). Try it for other triangles as well, and then make a conjecture for what happens in general! We will come back to why this happens in a later year.



(a) (b) (c)

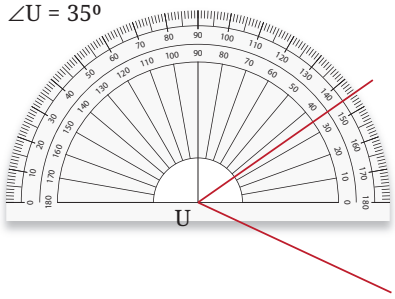
Fig. 2.21

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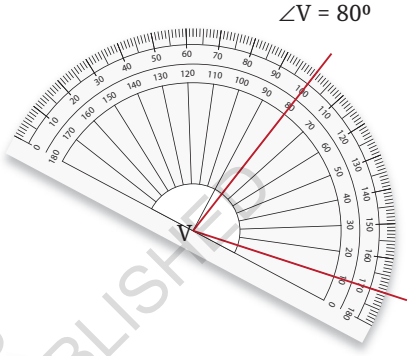
Mind the Mistake, Mend the Mistake!

A student used a protractor to measure the angles as shown below. In each figure, identify the incorrect usage(s) of the protractor and discuss how the reading could have been made and think how it can be corrected.

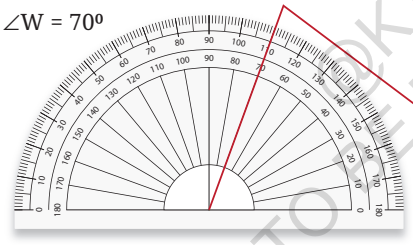
∠U = 35⁰



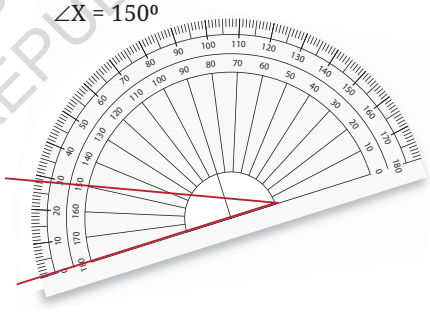
∠V = 80⁰



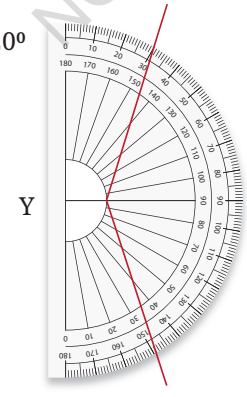
∠W = 70⁰



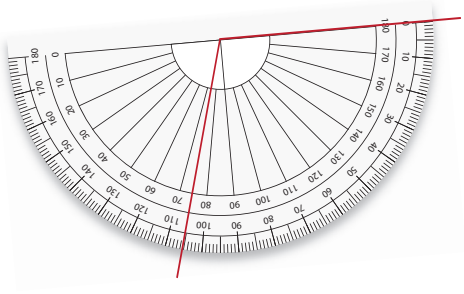
∠X = 150⁰



∠Y = 120⁰



∠Z = 85⁰



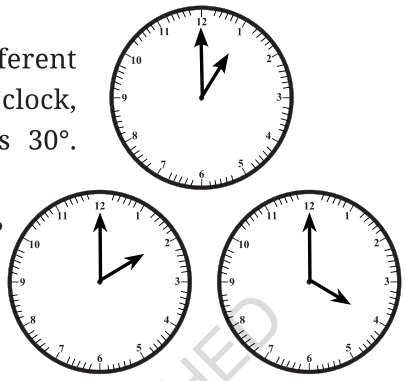
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Figure it Out

Where are the angles?

1. Angles in a clock:



   a. The hands of a clock make different angles at different times. At 1 o'clock, the angle between the hands is 30°. Why?

   b. What will be the angle at 2 o'clock? And at 4 o'clock? 6 o'clock?

   c. Explore other angles made by the hands of a clock.

2. The angle of a door:

   Is it possible to express the amount by which a door is opened using an angle? What will be the vertex of the angle and what will be the arms of the angle?



3. Vidya is enjoying her time on the swing. She notices that the greater the angle with which she starts the swinging, the greater is the speed she achieves on her swing. But where is the angle? Are you able to see any angle?



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4. Here is a toy with slanting slabs attached to its sides; the greater the angles or slopes of the slabs, the faster the balls roll. Can angles be used to describe the slopes of the slabs? What are the arms of each angle? Which arm is visible and which is not?



5. Observe the images below where there is an insect and its rotated version. Can angles be used to describe the amount of rotation? How? What will be the arms of the angle and the vertex?

   Hint: Observe the horizontal line touching the insects.



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Teacher's Note

It is important that students see the application of each mathematical concept in their daily lives. Teacher can organise some activities where students can appreciate the practical applications of angles in real-life situations, e.g., clocks, doors, swings, concepts of uphill and downhill, location of the sun, the giving of directions, etc.

2.10 Drawing Angles

Vidya wants to draw a 30° angle and name it ∠TIN using a protractor.

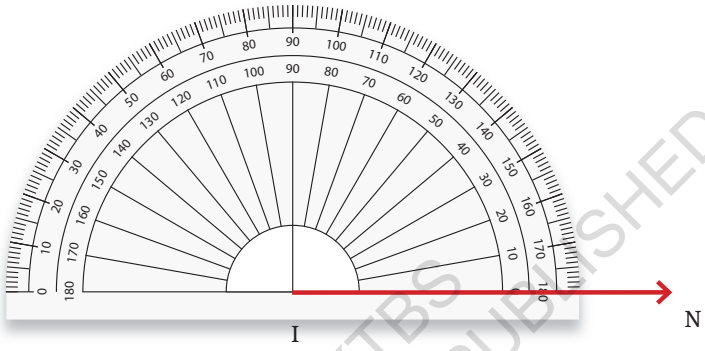
In ∠TIN, I will be the vertex, IT and IN will be the arms of the angle. Keeping one arm, say IN, as the reference (base), the other arm IT should take a turn of 30°.

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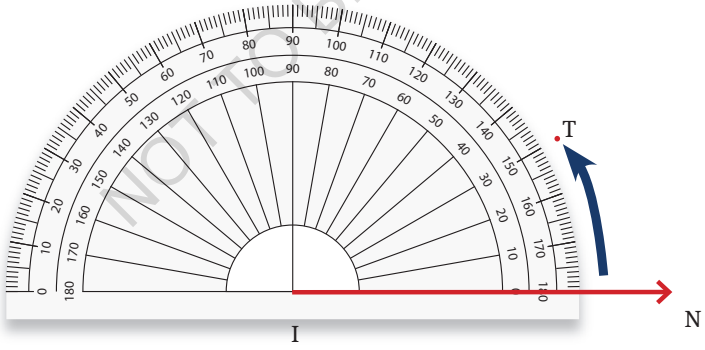
Step 1: We begin with the base and draw $\overrightarrow{IN}$:



Step 2: We will place the centre point of the protractor on I and align IN to the 0 line.



Step 3: Now, starting from 0, count your degrees (0, 10, 30) up to 30 on the protractor. Mark point T at the label 30°.

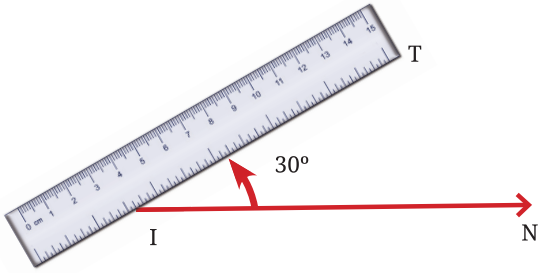


Step 4: Using a ruler join the point I and T.

$\angle TIN = 30°$ is the required angle.

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Let's Play a Game #1

This is an angle guessing game! Play this game with your classmates by making two teams, Team 1 and Team 2. Here are the instructions and rules for the game:

- **Team 1** secretly choose an angle measure, for example, 49° and makes an angle with that measure using a protractor without Team 2 being able to see it.
- **Team 2** now gets to look at the angle. They have to quickly discuss and guess the number of degrees in the angle (without using a protractor!).
- **Team 1** now demonstrates the true measure of the angle with a protractor.
- **Team 2** scores the number of points that is the absolute difference in degrees between their guess and the correct measure. For example, if Team 2 guesses 39°, then they score 10 points (49°–39°).
- Each team gets five turns. The winner is the team with the lowest score!

Let's Play a Game #2

We now change the rules of the game a bit. Play this game with your classmates by again making two teams, Team 1 and Team 2. Here are the instructions and rules:

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- **Team 1** announces to all, an angle measure, e.g., 34°.
- A player from **Team 2** must draw that angle on the board without using a protractor. Other members of **Team 2** can help the player by speaking words like 'Make it bigger!' or 'Make it smaller!'.
- A player from **Team 1** measures the angle with a protractor for all to see.
- **Team 2** scores the number of points that is the absolute difference in degrees between **Team 2**'s angle size and the intended angle size. For example, if player's angle from **Team 2** is measured to be 25°, then **Team 2** scores 9 points (34°–25°).
- Each team gets five turns. The winner is again the team with the lowest score.

Teacher's Note

These games are important to play to build intuition about angles and their measures. Return to this game at least once or twice on different days to build practice in estimating angles. Note that these games can also be played between pairs of students.

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Figure it Out

1. In Fig. 2.23, list all the angles possible. Did you find them all? Now, guess the measures of all the angles. Then, measure the angles with a protractor. Record all your numbers in a table. See how close your guesses are to the actual measures.

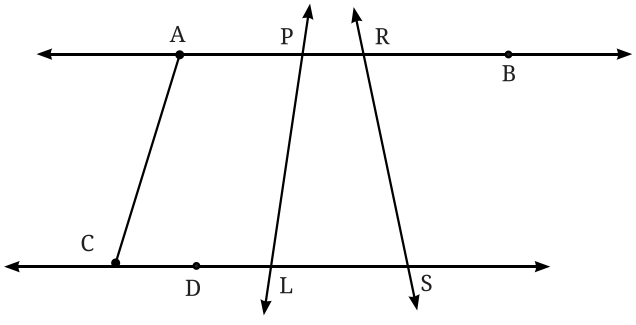
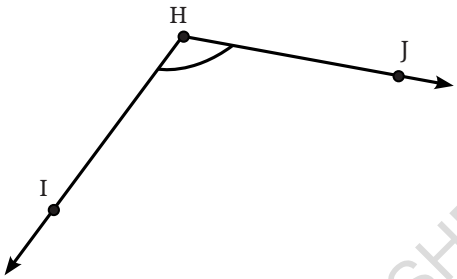


Fig. 2.23

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2. Use a protractor to draw angles having the following degree measures:
   a. 110° b. 40° c. 75° d. 112° e. 134°

3. Draw an angle whose degree measure is the same as the angle given below:



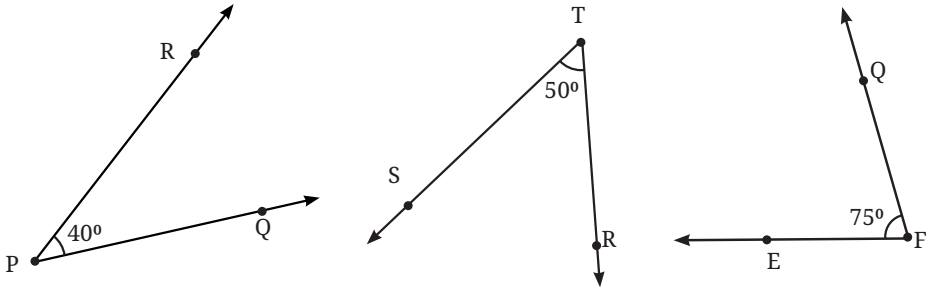
Also, write down the steps you followed to draw the angle.

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2.11 Types of Angles and their Measures

We have read about different types of angles in this chapter. We have seen that a straight angle is 180° and a right angle is 90°. How can other types of angles—acute and obtuse—be described in terms of their degree measures?

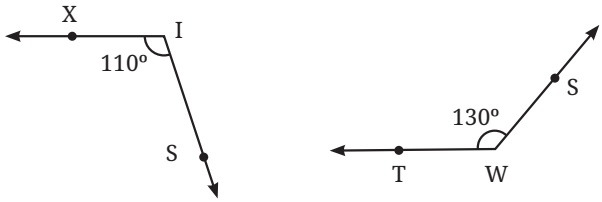
Acute Angle: Angles that are smaller than the right angle, i.e., less than 90° and are greater than 0°, are called **acute** angles.



Examples of acute angles

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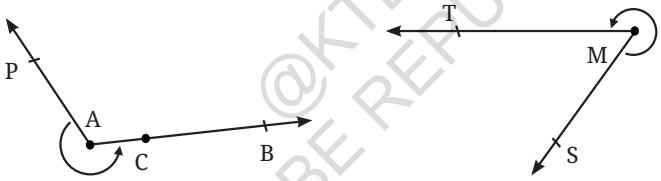
Obtuse Angle: Angles that are greater than the right angle and less than the straight angle, i.e., greater than 90° and less than 180°, are called **obtuse** angles.



Examples of obtuse angles

Have we covered all the possible measures that an angle can take? Here is another type of angle.

Reflex angle: Angles that are greater than the straight angle and less than the whole angle, i.e., greater than 180° and less than 360°, are called **reflex** angles.



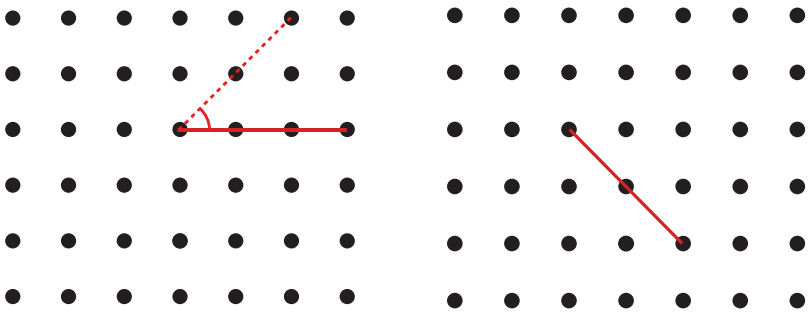
Examples of reflex angles

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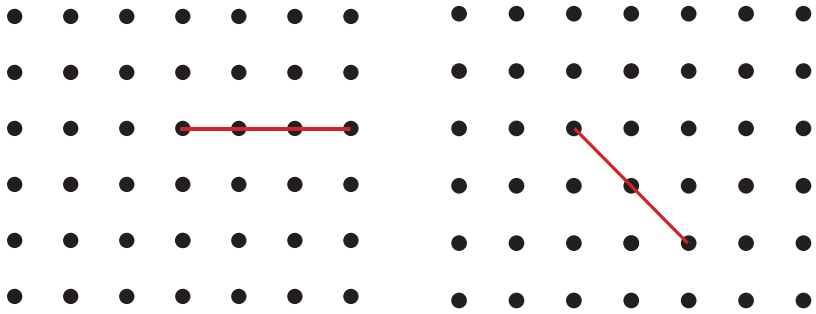
Figure it Out

1. In each of the below grids, join A to other grid points in the figure by a straight line to get:

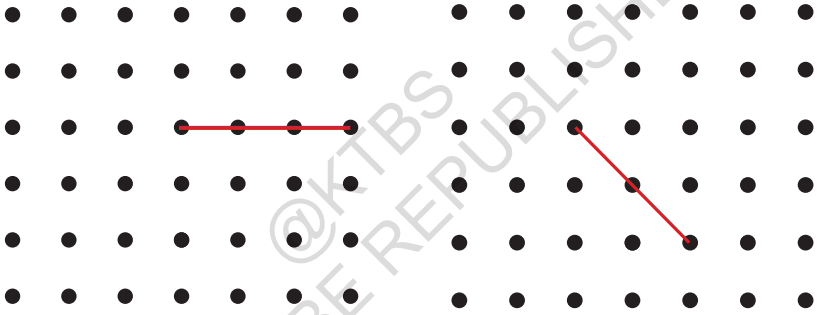
   a. An acute angle



b. An obtuse angle



c. A reflex angle

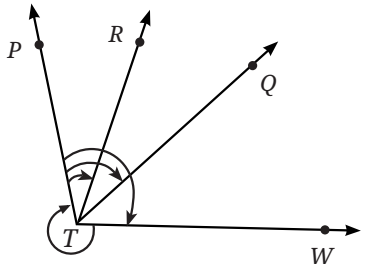


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Mark the intended angles with curves to specify the angles. One has been done for you.

2. Use a protractor to find the measure of each angle. Then classify each angle as acute, obtuse, right, or reflex.

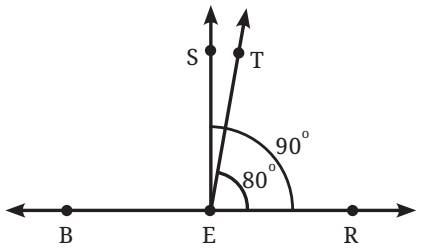
   a. ∠PTR b. ∠PTQ c. ∠PTW d. ∠WTP



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Let's Explore

In this figure, ∠TER = 80°. What is the measure of ∠BET? What is the measure of ∠SET?



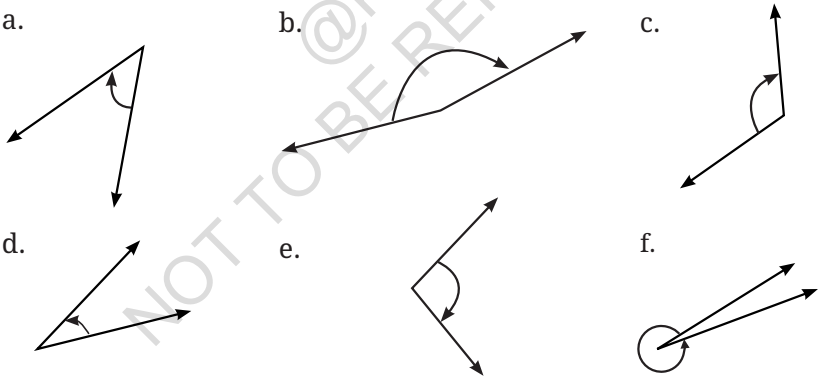
Hint: Observe that ∠REB is a straight angle. Hence, the degree measure of ∠REB = 180° of which 80° is covered by ∠TER. A similar argument can be applied to find the measure of ∠SET.

Figure it Out

1. Draw angles with the following degree measures:

   a. 140° b. 82° c. 195° d. 70° e. 35°

2. Estimate the size of each angle and then measure it with a protractor:



   Classify these angles as acute, right, obtuse or reflex angles.

3. Make any figure with three acute angles, one right angle and two obtuse angles.
4. Draw the letter 'M' such that the angles on the sides are 40° each and the angle in the middle is 60°.
5. Draw the letter 'Y' such that the three angles formed are 150°, 60° and 150°.

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6. The *Ashoka Chakra* has 24 spokes. What is the degree measure of the angle between two spokes next to each other? What is the largest acute angle formed between two spokes?



7. **Puzzle:** I am an acute angle. If you double my measure, you get an acute angle. If you triple my measure, you will get an acute angle again. If you quadruple (four times) my measure, you will get an acute angle yet again! But if you multiply my measure by 5, you will get an obtuse angle measure. What are the possibilities for my measure?

SUMMARY

- A **point** determines a location. It is denoted by a capital letter.
- A **line segment** corresponds to the shortest distance between two points. The line segment joining points S and T is denoted by $\overline{ST}$.
- A **line** is obtained when a line segment like $\overline{ST}$ is extended on both sides indefinitely; it is denoted by $\overleftrightarrow{ST}$ or sometimes by a single small letter like *m*.
- A **ray** is a portion of a line starting at a point D and going in one direction indefinitely. It is denoted by $\overrightarrow{DP}$ where P is another point on the ray.
- An angle can be visualised as two rays starting from a common starting point. Two rays $\overrightarrow{OP}$ and $\overrightarrow{OM}$ form the angle $\angle POM$ (also called $\angle MOP$); here, O is called the **vertex** of the angle, and the rays $\overrightarrow{OP}$ and $\overrightarrow{OM}$ are called the **arms** of the angle.
- The size of an angle is the amount of rotation or turn needed about the vertex to rotate one ray of the angle onto the other ray of the angle.
- The sizes of angles can be measured in **degrees**. One full rotation or turn is considered as 360 degrees and denoted as 360°.
- Degree measures of angles can be measured using a **protractor**.
- Angles can be **straight** (180°), **right** (90°), **acute** (more than 0° and less than 90°), **obtuse** (more than 90° and less than 180°), and **reflex** (more than 180° and less than 360°).

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3 NUMBER PLAY

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Numbers are used in different contexts and in many different ways to organise our lives. We have used numbers to count, and have applied the basic operations of addition, subtraction, multiplication and division on them, to solve problems related to our daily lives.

In this chapter, we will continue this journey, by playing with numbers, seeing numbers around us, noticing patterns, and learning to use numbers and operations in new ways.

Think about various situations where we use numbers. List five different situations in which numbers are used. See what your classmates have listed, share, and discuss.

Math Talk

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3.1 Numbers can Tell us Things

What are these numbers telling us?

Some children in a park are standing in a line. Each one says a number.



What do you think these numbers mean?

The children now rearrange themselves, and again each one says a number based on the arrangement.



Did you figure out what these numbers represent?
Hint: Could their heights be playing a role?

A child says '1' if there is only one taller child standing next to them.
A child says '2' if both the children standing next to them are taller.
A child says '0', if neither of the children standing next to them are taller.
That is each person says the number of taller neighbours they have.

Try answering the questions below and share your reasoning.

Math Talk

1. Can the children rearrange themselves so that the children standing at the ends say '2'?
2. Can we arrange the children in a line so that all would say only 0s?
3. Can two children standing next to each other say the same number?
4. There are 5 children in a group, all of different heights. Can they stand such that four of them say '1' and the last one says '0'? Why or why not?
5. For this group of 5 children, is the sequence 1, 1, 1, 1, 1 possible?
6. Is the sequence 0, 1, 2, 1, 0 possible? Why or why not?
7. How would you rearrange the five children so that the maximum number of children say '2'?

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3.2 Supercells

Observe the numbers written in the table below. Why are some numbers coloured? Discuss.

43	79	75	63	10	29	28	34
200	577	626	345	790	694	109	198

A cell is coloured if the number in it is larger than its adjacent cells. The number 626 is coloured as it is larger than 577 and 345, whereas 200 is not coloured as it is smaller than 577. The number 198 is coloured as it has only one adjacent cell with 109 in it, and 198 is larger than 109.

Figure it Out

1. Colour or mark the supercells in the table below.

6828	670	9435	3780	3708	7308	8000	5583	52

2. Fill the table below with only 4-digit numbers such that the supercells are exactly the coloured cells.

5346			1258			9635	

3. Fill the table below such that we get as many supercells as possible. Use numbers between 100 and 1000 without repetitions.

4. Out of the 9 numbers, how many supercells are there in the table above? _____

5. Find out how many supercells are possible for different numbers of cells.

   Do you notice any pattern? What is the method to fill a given table to get the maximum number of supercells? Explore and share your strategy.

Math Talk

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6. Can you fill a supercell table without repeating numbers such that there are no supercells? Why or why not?
7. Will the cell having the largest number in a table always be a supercell? Can the cell having the smallest number in a table be a supercell? Why or why not?
8. Fill a table such that the cell having the second largest number is not a supercell.
9. Fill a table such that the cell having the second largest number is not a supercell but the second smallest number is a supercell. Is it possible?
10. Make other variations of this puzzle and challenge your classmates.

Let's do the supercells activity with more rows.

Here the neighbouring cells are those that are immediately to the left, right, top and bottom.

The rule remains the same: a cell becomes a supercell if the number in it is greater than all the numbers in its neighbouring cells. In Table 1, 8632 is greater than all its neighbours 4580, 8280, 4795 and 1944.

Table 1

2430	7500	7350	9870
3115	4795	9124	9230
4580	8632	8280	3446
5785	1944	5805	6034

Complete Table 2 with 5-digit numbers whose digits are '1', '0', '6', '3', and '9' in some order. Only a coloured cell should have a number greater than all its neighbours.

The biggest number in the table is _____ .

Table 2

	96,301	36,109	
	13,609	60,319	19,306
		60,193	
	10,963		

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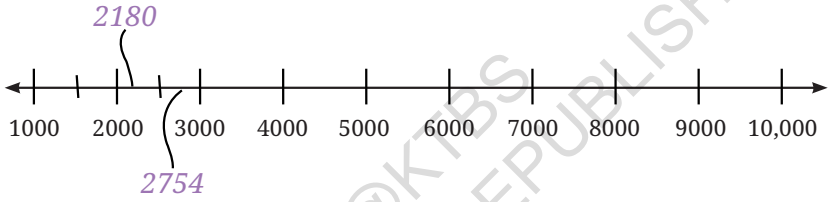
The smallest even number in the table is _____.

The smallest number greater than 50,000 in the table is _____.

Once you have filled the table above, put commas appropriately after the thousands digit.

3.3 Patterns of Numbers on the Number Line

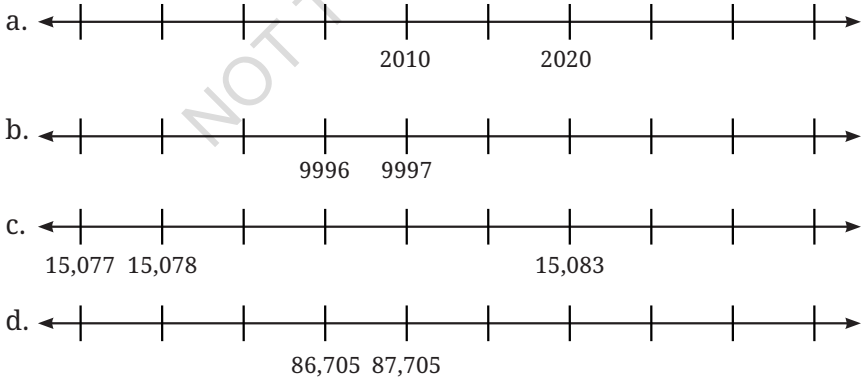
We are quite familiar with number lines now. Let's see if we can place some numbers in their appropriate positions on the number line. Here are the numbers: 2180, 2754, 1500, 3600, 9950, 9590, 1050, 3050, 5030, 5300 and 8400.



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Figure it Out

Identify the numbers marked on the number lines below, and label the remaining positions.



Put a circle around the smallest number and a box around the largest number in each of the sequences above.

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3.4 Playing with Digits

We start writing numbers from 1, 2, 3 ... and so on. There are nine 1-digit numbers.

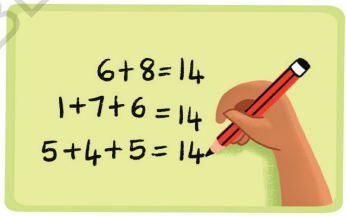
Find out how many numbers have two digits, three digits, four digits, and five digits.

1-digit numbers From 1–9	2-digit numbers	3-digit numbers	4-digit numbers	5-digit numbers
9				

Digit sums of numbers

Komal observes that when she adds up digits of certain numbers the sum is the same.

For example, adding the digits of the number 68 will be same as adding the digits of 176 or 545.



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Figure it Out

1. **Digit sum 14**
   a. Write other numbers whose digits add up to 14.
   b. What is the smallest number whose digit sum is 14?
   c. What is the largest 5-digit whose digit sum is 14?
   d. How big a number can you form having the digit sum of 14? Can you make an even bigger number?

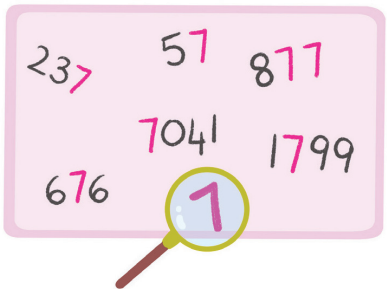
Math Talk

2. Find out the digit sums of all the numbers from 40 to 70. Share your observations with the class.
3. Calculate the digit sums of 3-digit numbers whose digits are consecutive (for example, 345). Do you see a pattern? Will this pattern continue?

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Digit Detectives

After writing numbers from 1 to 100, Dinesh wondered how many times he would have written the digit '7'!



Among the numbers 1–100, how many times will the digit '7' occur? Among the numbers 1–1000, how many times will the digit '7' occur?

3.5 Pretty Palindromic Patterns

What pattern do you see in these numbers: 66, 848, 575, 797, 1111? These numbers read the same from left to right and from right to left. Try and see. Such numbers are called **palindromes** or **palindromic numbers**.

All palindromes using 1, 2, 3

The numbers 121, 313, 222 are some examples of palindromes using the digits '1', '2', 3'.

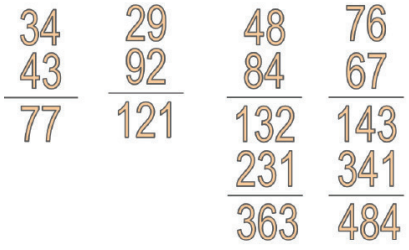
Write all possible 3-digit palindromes using these digits.

Reverse-and-add palindromes

Now, look at these additions. Try to figure out what is happening.

Steps to follow: Start with a 2-digit number. Add this number to its reverse. Stop if you get a palindrome or else repeat the steps of reversing the digits and adding.

Try the same procedure for some other numbers, and perform the same steps. Stop if



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you get a palindrome. There are numbers for which you have to repeat this a large number of times.

Are there numbers for which you do not reach a palindrome at all?

Explore

Will reversing and adding numbers repeatedly, starting with a 2-digit number, always give a palindrome? Explore and find out.*

Math Talk

Puzzle time

tth	th	h	t	u

Write the number in words:

I am a 5-digit palindrome.
I am an odd number.
My 't' digit is double of my 'u' digit.
My 'h' digit is double of my 't' digit.
Who am I? _____

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3.6 The Magic Number of Kaprekar



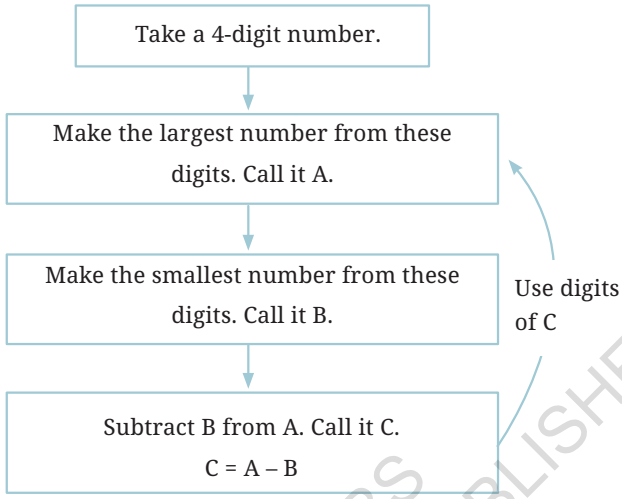
D.R. Kaprekar was a mathematics teacher in a government school in Devlali, Maharashtra. He liked playing with numbers very much and found many beautiful patterns in numbers that were previously unknown.

In 1949, he discovered a fascinating and magical phenomenon when playing with 4-digit numbers.

**The answer is yes! For 3-digit numbers the answer is unknown. It is suspected that starting with 196 never yields a palindrome!*

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Follow these steps and experience the magic for yourselves! Pick any 4-digit number having at least two different digits, say 6382.



What happens if we continue doing this?

A = 8632
B = 2368
C = 8632–2368
= 6264

A = 6642
B = 2466
C = 6642–2466
= 4176

A = 7641
B = 1467
C = 7641–1467
= 6174

A =
B =
C =

Explore

Take different 4-digit numbers and try carrying out these steps. Find out what happens. Check with your friends what they got.

You will always reach the magic number '6174'! The number '6174' is now called the 'Kaprekar constant'.

Carry out these same steps with a few 3-digit numbers. What number will start repeating?

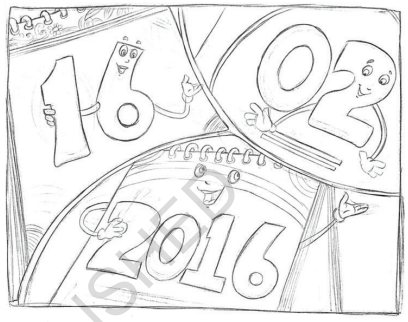
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3.7 Clock and Calendar Numbers

On the usual 12-hour clock, there are timings with different patterns. For example, 4:44, 10:10, 12:21.

Try and find out all possible times on a 12-hour clock of each of these types.



Manish has his birthday on 20/12/2012 where the digits '2', '0', '1', and '2' repeat in that order.

Find some other dates of this form from the past.

His sister, Meghana, has her birthday on 11/02/2011 where the digits read the same from left to right and from right to left.

Find all possible dates of this form from the past.

Jeevan was looking at this year's calendar. He started wondering, "Why should we change the calendar every year? Can we not reuse a calendar?". What do you think?

You might have noticed that last year's calendar was different from this year's. Also, next year's calendar will also be different from the previous years.

But, will any year's calendar repeat again after some years? Will all dates and days in a year match exactly with that of another year?

Try This

Figure it Out

1. Pratibha uses the digits '4', '7', '3' and '2', and makes the smallest and largest 4-digit numbers with them: 2347 and 7432. The difference between these two numbers is 7432 – 2347 = 5085. The sum of these two numbers is 9779. Choose 4-digits to make:
   a. the difference between the largest and smallest numbers greater than 5085.

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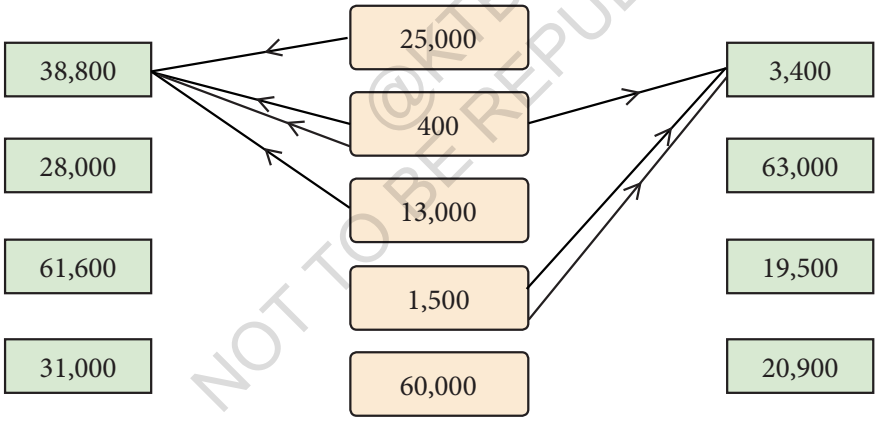
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b. the difference between the largest and smallest numbers less than 5085.
c. the sum of the largest and smallest numbers greater than 9779.
d. the sum of the largest and smallest numbers less than 9779.

2. What is the sum of the smallest and largest 5-digit palindrome? What is their difference?
3. The time now is 10:01. How many minutes until the clock shows the next palindromic time? What about the one after that?
4. How many rounds does the number 5683 take to reach the Kaprekar constant?

3.8 Mental Math

Observe the figure below. What can you say about the numbers and the lines drawn?



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Numbers in the middle column are added in different ways to get the numbers on the sides (1500 + 1500 + 400 = 3400). The numbers in the middle can be used as many times as needed to get the desired sum. Draw arrows from the middle to the numbers on the sides to obtain the desired sums.
Two examples are given. It is simpler to do it mentally!

38,800 = 25,000 + 400 × 2 + 13,000
3400 = 1500 + 1500 + 400

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Can we make 1,000 using the numbers in the middle? Why not? What about 14,000, 15,000 and 16,000? Yes, it is possible. Explore how. What thousands cannot be made?



Adding and Subtracting

Here, using the numbers in the boxes, we are allowed to use both addition and subtraction to get the required number. An example is shown.

40,000	7,000
300	1,500
12,000	800

39,800 = 40,000 – 800 + 300 + 300

45,000 =

5,900 =

17,500 =

21,400 =

Digits and Operations

An example of adding two 5-digit numbers to get another 5-digit number is 12,350 + 24,545 = 36,895.

An example of subtracting two 5-digit numbers to get another 5-digit number is 48,952 – 24,547 = 24,405.

Figure it Out

1. Write an example for each of the below scenarios whenever possible.

5-digit + 5-digit to give a 5-digit sum more than 90,250	5-digit + 3-digit to give a 6-digit sum	4-digit + 4-digit to give a 6-digit sum	5-digit + 5-digit to give a 6-digit sum	5-digit + 5-digit to give 18,500
5-digit – 5-digit to give a difference less than 56,503	5-digit – 3-digit to give a 4-digit difference	5-digit – 4-digit to give a 4-digit difference	5-digit – 5-digit to give a 3-digit difference	5-digit – 5-digit to give 91,500

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Could you find examples for all the cases? If not, think and discuss what could be the reason. Make other such questions and challenge your classmates.

Math Talk

2. *Always, Sometimes, Never?*

Below are some statements. Think, explore and find out if each of the statement is 'Always true', 'Only sometimes true' or 'Never true'. Why do you think so? Write your reasoning and discuss this with the class.

a. 5-digit number + 5-digit number gives a 5-digit number

b. 4-digit number + 2-digit number gives a 4-digit number

c. 4-digit number + 2-digit number gives a 6-digit number

d. 5-digit number – 5-digit number gives a 5-digit number

e. 5-digit number – 2-digit number gives a 3-digit number

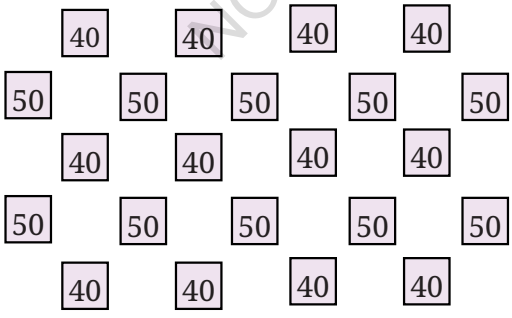
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3.9 Playing with Number Patterns

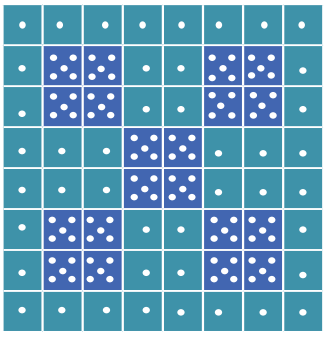
Here are some numbers arranged in some patterns. Find out the sum of the numbers in each of the below figures. Should we add them one by one or can we use a quicker way?

Share and discuss in class the different methods each one of you used to solve these questions.

a.



b.

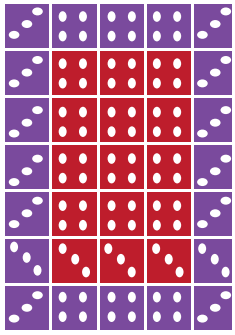


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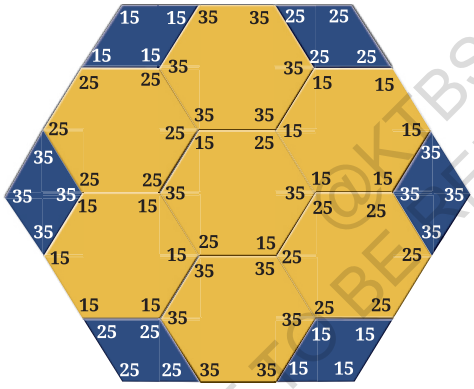
c.

32	32	32	32	32	32	32	32
32	32	32	32	32	32	32	32
32	32	32	32	32	32	32	32
32	32	32	32	32	32	32	32
64	64	64					64
64	64	64					64
64	64	64					64
64	64	64					64

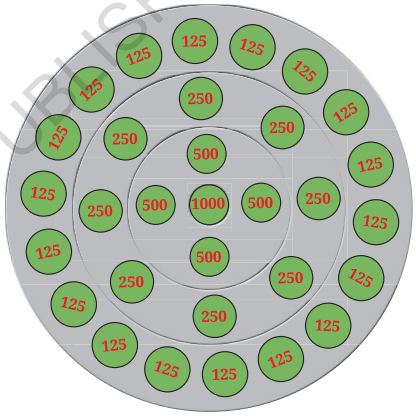
d.



e.



f.



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3.10 An Unsolved Mystery—the Collatz Conjecture!

Look at the sequences below—the same rule is applied in all the sequences:

a. 12, 6, 3, 10, 5, 16, 8, 4, 2, 1

b. 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

c. 21, 64, 32, 16, 8, 4, 2, 1

d. 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

Do you see how these sequences were formed?

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The rule is: one starts with any number; if the number is even, take half of it; if the number is odd, multiply it by 3 and add 1; repeat.

Notice that all four sequences above eventually reached the number 1. In 1937, the German mathematician, Lothar Collatz conjectured that the sequence will *always* reach 1, regardless of the whole number you start with. Even today—despite many mathematicians working on it — it remains an unsolved problem as to whether Collatz's conjecture is true! Collatz's conjecture is one of the most famous unsolved problems in mathematics.

Make some more Collatz sequences like those above, starting with your favourite whole numbers. Do you always reach 1?

Do you believe the conjecture of Collatz that all such sequences will eventually reach 1? Why or why not?

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3.11 Simple Estimation

At times, we may not know or need an exact count of things and an estimate is sufficient for the purpose at hand. For example, your school headmaster might know the exact number of students enrolled in your school, but you may only know an estimated count. How many students are in your school? About 150? 400? A thousand?

Paromita's class section has 32 children. The other 2 sections of her class have 29 and 35 children. So, she estimated the number of children in her class to be about 100. Along with Class 6, her school also has Classes 7–10 and each class has 3 sections each. She assumed a similar number in each class and estimated the number of students in her school to be around 500.

Figure it Out

We shall do some simple estimates. It is a fun exercise, and you may find it amusing to know the various numbers around us. Remember,

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we are not interested in the exact numbers for the following questions. Share your methods of estimation with the class.

1. Steps you would take to walk:
   a. From the place you are sitting to the classroom door
   b. Across the school ground from start to end
   c. From your classroom door to the school gate
   d. From your school to your home
2. Number of times you blink your eyes or number of breaths you take:
   a. In a minute
   b. In an hour
   c. In a day
3. Name some objects around you that are:
   a. a few thousand in number
   b. more than ten thousand in number

Estimate the answer

Try to guess within 30 seconds. Check your guess with your friends.

1. Number of words in your maths textbook:
   a. More than 5000
   b. Less than 5000
2. Number of students in your school who travel to school by bus:
   a. More than 200
   b. Less than 200
3. Roshan wants to buy milk and 3 types of fruit to make fruit custard for 5 people. He estimates the cost to be ₹100. Do you agree with him? Why or why not?
4. Estimate the distance between Gandhinagar (in Gujarat) to Kohima (in Nagaland).
   Hint: Look at the map of India to locate these cities.

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5. Sheetal is in Grade 6 and says she has spent around 13,000 hours in school till date. Do you agree with her? Why or why not?
6. Earlier, people used to walk long distances as they had no other means of transport. Suppose you walk at your normal pace. Approximately, how long would it take you to go from:
   a. Your current location to one of your favourite places nearby.
   b. Your current location to any neighbouring state's capital city.
   c. The southernmost point in India to the northernmost point in India.
7. Make some estimation questions and challenge your classmates!

3.12 Games and Winning Strategies

Numbers can also be used to play games and develop winning strategies.

Here is a famous game called 21. Play it with a classmate. Then try it at home with your family!

Rules for Game #1: The first player says 1, 2 or 3. Then the two players take turns adding 1, 2, or 3 to the previous number said. The first player to reach 21 wins!

Play this game several times with your classmate. Are you starting to see the winning strategy?

Which player can always win if they play correctly? What is the pattern of numbers that the winning player should say?

There are many variations of this game. Here is another common variation:

Rules for Game #2: The first player says a number between 1 and 10. Then the two players take turns adding a number between 1 and 10 to the previous number said. The first player to reach 99 wins!

Play this game several times with your classmate. See if you can figure out the corresponding winning strategy in this case! Which

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player can always win? What is the pattern of numbers that the winning player should say this time?

Make your own variations of this game — decide how much one can add at each turn, and what number is the winning number. Then play your game several times, and figure out the winning strategy and which player can always win!

Figure it Out

1. There is only one supercell (number greater than all its neighbours) in this grid. If you exchange two digits of one of the numbers, there will be 4 supercells. Figure out which digits to swap.

16,200	39,344	29,765
23,609	62,871	45,306
19,381	50,319	38,408

Try This

2. How many rounds does your year of birth take to reach the Kaprekar constant?
3. We are the group of 5-digit numbers between 35,000 and 75,000 such that all of our digits are odd. Who is the largest number in our group? Who is the smallest number in our group? Who among us is the closest to 50,000?
4. Estimate the number of holidays you get in a year including weekends, festivals and vacation. Then, try to get an exact number and see how close your estimate is.
5. Estimate the number of liters a mug, a bucket and an overhead tank can hold.
6. Write one 5-digit number and two 3-digit numbers such that their sum is 18,670.
7. Choose a number between 210 and 390. Create a number pattern similar to those shown in Section 3.9 that will sum up to this number.

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8. Recall the sequence of Powers of 2 from Chapter 1, Table 1. Why is the Collatz conjecture correct for all the starting numbers in this sequence?
9. Check if the Collatz Conjecture holds for the starting number 100.
10. Starting with 0, players alternate adding numbers between 1 and 3. The first person to reach 22 wins. What is the winning strategy now?

SUMMARY

- Numbers can be used for many different purposes including, to convey information, make and discover patterns, estimate magnitudes, pose and solve puzzles, and play and win games.
- Thinking about and formulating set procedures to use numbers for these purposes is a useful skill and capacity (called 'computational thinking').
- Many problems about numbers can be very easy to pose, but very difficult to solve. Indeed, numerous such problems are still unsolved (for example, Collatz's Conjecture).

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4 DATA HANDLING AND PRESENTATION

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If you ask your classmates about their favourite colours, you will get a list of colours. This list is an example of data. Similarly, if you measure the weight of each student in your class, you would get a collection of measures of weight—again data.

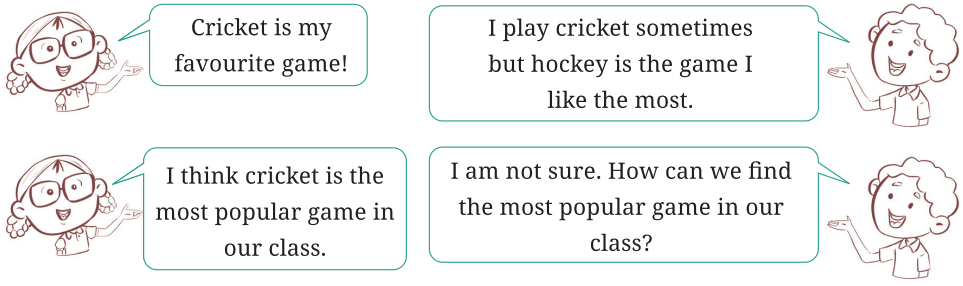
Any collection of facts, numbers, measures, observations or other descriptions of things that convey *information* about those things is called **data**.

We live in an age of information. We constantly see large amounts of data presented to us in new and interesting ways. In this chapter, we will explore some of the ways that data is presented, and how we can use some of those ways to correctly display, interpret and make inferences from such data!

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4.1 Collecting and Organising Data

Navya and Naresh are discussing their favourite games.





To figure out the most popular game in their class, what should Navya and Naresh do? Can you help them?

Naresh and Navya decided to go to each student in the class and ask what their favourite game is. Then they prepared a list.

Navya is showing the list:



Mehnoor – *Kabaddi*	Pushkal – *Satoliya* (*Pittu*)	Anaya – *Kabaddi*
Jubimon – Hockey	Densy – Badminton	Jivisha – *Satoliya* (*Pittu*)
Simran – *Kabaddi*	Jivika – *Satoliya* (*Pittu*)	Rajesh – Football
Nand – *Satoliya* (*Pittu*)	Leela – Hockey	Thara – Football
Ankita – *Kabaddi*	Afshan – Hockey	Soumya – Cricket
Imon – Hockey	Keerat – Cricket	Navjot – Hockey
Yuvraj – Cricket	Gurpreet – Hockey	Hemal – *Satoliya* (*Pittu*)
Rehana – Hockey	Arsh – *Kabaddi*	Debabrata – Football
Aarna – Badminton	Bhavya – Cricket	Ananya – Hockey
Kompal – Football	Sarah – *Kabaddi*	Hardik – Cricket
Tahira – Cricket		

She says (happily), "I have collected the data. I can figure out the most popular game now!".

A few other children are looking at the list and wondering, "We can't yet see the most popular game. How can we get it from this list?".

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Figure it Out

1. What would you do to find the most popular game among Naresh's and Navya's classmates?
2. What is the most popular game in their class?
3. Try to find out the most popular game among your classmates.
4. Pari wants to respond to the questions given below. Put a tick (✓) for the questions where she needs to carry out data collection and

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put a cross (×) for the questions where she doesn't need to collect data. Discuss your answers in the classroom.

a. What is the most popular TV show among her classmates? ☐

b. When did India get independence? ☐

c. How much water is getting wasted in her locality? ☐

d. What is the capital of India? ☐

Shri Nilesh is a teacher. He decided to bring sweets to the class to celebrate the new year. The sweets shop nearby has *jalebi*, *gulab jamun*, *gujiya*, *barfi*, and *rasgulla*. He wanted to know the choices of the children. He wrote the names of the sweets on the board and asked each child to tell him their preference. He put a tally mark '|' for each student and when the count reached 5, he put a line through the previous four and marked it as 𝍸.

Sweets	Tally Marks	No. of Students
Jalebi	𝍸 |	6
Gulab jamun	𝍸 ||||	9
Gujiya	𝍸 𝍸 |||	_____
Barfi	|||	_____
Rasgulla	𝍸 ||	_____

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Figure it Out

1. Complete the table to help Shri Nilesh to purchase the correct numbers of sweets:

   a. How many students chose jalebi? ☐

   b. Barfi was chosen by ☐ students?

   c. How many students chose gujiya? ☐

   d. Rasgulla was chosen by ☐ students?

   e. How many students chose gulab jamun? ☐

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Shri Nilesh requested one of the staff members to bring the sweets as given in the table. The above table helped him to purchase the correct numbers of sweets.

2. Is the above table sufficient to distribute each type of sweet to the correct student? Explain. If it is not sufficient, what is the alternative?

To organise the data, we can write the name of each sweet in one column and using tally signs, note the number of students who prefer that sweet. The numbers 6, 9, ... are the **frequencies** of the sweet preferences for *jalebi, gulab jamun* ... respectively.

Sushri Sandhya asked her students about the sizes of the shoes they wear. She noted the data on the board.

4	5	3	4	3	4	5	5	4
5	5	4	5	6	4	3	5	6
4	6	4	5	7	5	6	4	5

She then arranged the shoe sizes of the students in ascending order —

3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 7

Figure it Out

1. Help her to figure out the following:
   a. The largest shoe size in the class is _____.
   b. The smallest shoe size in the class is _____.
   c. There are _____ students who wear shoe size 5.
   d. There are _____ students who wear shoe sizes larger than 4.
2. How did arranging the data in ascending order help to answer these questions?
3. Are there other ways to arrange the data?



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4. Write the names of a few trees you see around you. When you observe a tree on the way from your home to school (or while walking from one place to another place), record the data and fill in the following table:

Tree	No. of Trees
Peepal	
Neem	
...	
....	

a. Which tree was found in the greatest number?

b. Which tree was found in the smallest number?

c. Were there any two trees found in the same numbers?

5. Take a blank piece of paper and paste any small news item from a newspaper. Each student may use a different article. Now, prepare a table on the piece of paper as given below. Count the number of each of the letters 'c', 'e', 'i', 'r', and 'x' in the words of the news article, and fill in the table.

Letter	c	e	i	r	x	Any other letter of your choice
Number of times found in the news item						

a. The letter found the most number of times is _____.

b. The letter found the least number of times is _____.

c. List the five letters 'c', 'e', 'i', 'r', 'x' in ascending order of frequency. Now, compare the order of your list with that of your classmates. Is your order the same or nearly the same as theirs? (Almost everyone is likely to get the order 'x, c, r, i, e'.) Why do you think this is the case?

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d. Write the process you followed to complete this task.

e. Discuss with your friends the processes they followed.

f. If you do this task with another news item, what process would you follow?

Teacher's Note

Provide more opportunities to collect and organise data. Ask students to guess what is the most popular colour, game, toy, school subject, etc., amongst the students in their classroom and then collect the data for it. It can be a fun activity in which they also learn about their classmates. Discuss how they can organise the data in different ways, each way having its own advantages and limitations. For all these tasks and the tasks under 'Figure it Out', discuss the tasks with the children and let them understand the tasks, and then let them plan and present their research processes and conclusions in the class.

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4.2 Pictographs

Pictographs are one visual and suggestive way to represent data without writing any numbers. Look at this picture — you may be familiar with it from previous classes.

Modes of Travelling	Number of Students ☺ = 1 Student
Private car	☺☺☺☺
Public bus	☺☺☺☺☺
School bus	☺☺☺☺☺☺☺☺☺☺☺
Cycle	☺☺☺
Walking	☺☺☺☺☺☺☺

This picture helps you understand at a glance the different modes of travel used by students. Based on this picture, answer the following questions:

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- Which mode of travel is used by the most number of students?
- Which mode of travel is used by the least number of students?

A pictograph represents data through pictures of objects. It helps answer questions about data with just a quick glance.

In the above pictograph, one unit or symbol (☺) is used to represent one student. There are also other pictographs where one unit or symbol stands for many people or objects.

Example: Nand Kishor collected responses from the children of his middle school in Berasia regarding how often they slept at least 9 hours during the night. He prepared a pictograph from the data:

Response	Number of Children (= 10 Children)
Always	
Sometimes	
Never	

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Answer the following questions using the pictograph:

1. What is the number of children who always slept at least 9 hours at night?
2. How many children sometimes slept at least 9 hours at night?
3. How many children always slept less than 9 hours each night? Explain how you got your answer.

Solutions

1. In the table, there are 5 pictures for 'Always'. Each picture represents 10 children. Therefore, 5 pictures indicate $5 \times 10 = 50$ children.
2. There are 2 complete pictures ($2 \times 10 = 20$) and a half picture (half of $10 = 5$). Therefore, the number of children who sleep at least 9 hours only sometimes is $20 + 5 = 25$.

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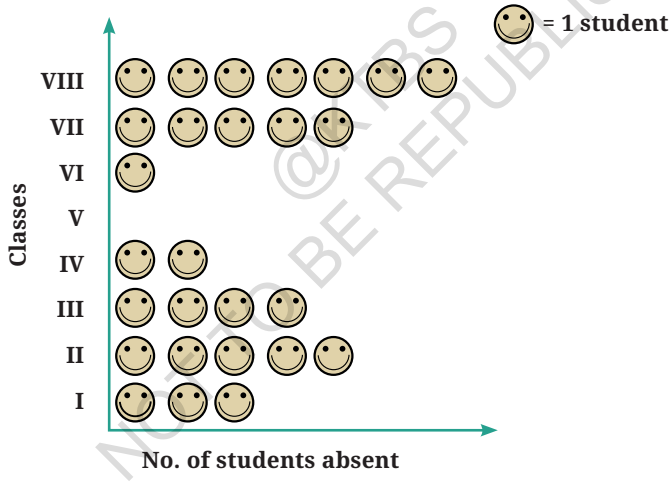
3. There are 4 complete pictures for 'Never'. Hence, 4 × 10 = 40 children never sleep at least 9 hours in a night, i.e., they always sleep less than 9 hours.

Drawing a Pictograph

One day, Lakhanpal collected data on how many students were absent in each class:

Class	1	2	3	4	5	6	7	8
No. of students	3	5	4	2	0	1	5	7

He created a pictograph to present this data and decided to show 1 student as ☺ in the pictograph —



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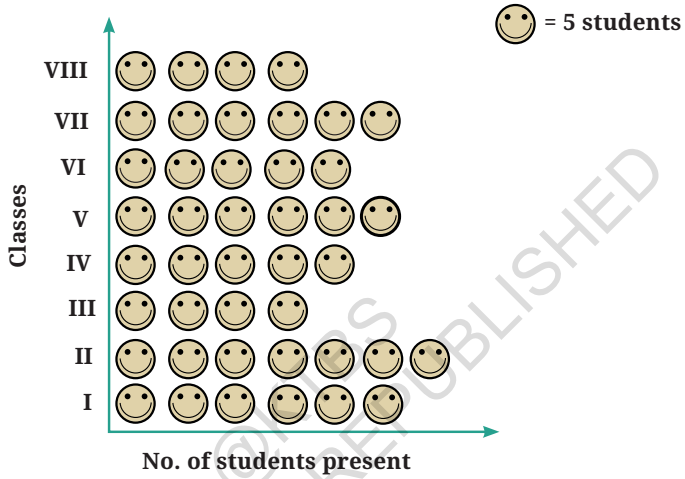
Meanwhile, his friends Jarina and Sangita collected data on how many students were present in each class:

Class	1	2	3	4	5	6	7	8
No. of students	30	35	20	25	30	25	30	20

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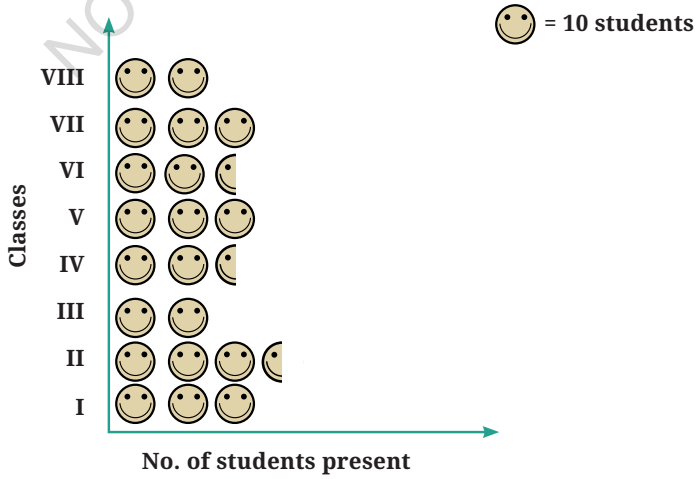
If they want to show their data through a pictograph, where they also use one symbol for each student, as Lakhanpal did, what are the challenges they might face?

Jarina made a plan to address this — since there were many students, she decided to use to represent 5 students. She figured that would save time and space too.



Sangita decided to use one to represent 10 students instead.

Since she used one to show 10 students, she had a problem in showing 25 students and 35 students in the pictograph. Then, she realised she could use to show 5 students.



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What could be the problems faced in preparing such a pictograph, if the total number of students present in a class is 33 or 27?



- **Pictographs** are a nice visual and suggestive way to represent data. They represent data through pictures of objects.
- Pictographs can help answer questions and make inferences about data with just a quick glance (in the examples above—about favourite games, favourite colours, most common modes of conveyance, number of students absent, etc.).
- By reading a pictograph, we can quickly understand the frequencies of the different categories (for example, cricket, hockey, etc.) and the comparisons of these frequencies.
- In a pictograph, the categories can be arranged horizontlly or vertically. For each category, simple pictures and symbols are then drawn in the designated columns or rows according to the frequency of that category.
- A **scale** or **key** (for example, ☺ : 1 student or ☺ : 5 students) is added to show what each symbol or picture represents. Each symbol or picture can represent one unit or multiple units.
- It can be more challenging to prepare a pictograph when the amount of data is large or when the frequencies are not exact multiples of the scale or key.

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Figure it Out

1. The following pictograph shows the number of books borrowed by students, in a week, from the library of Middle School, Ginnori:

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Day	Number of Books Borrowed ( =1 Book)
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	

a. On which day were the minimum number of books borrowed?
b. What was the total number of books borrowed during the week?
c. On which day were the maximum number of books borrowed? What may be the possible reason?

2. Magan Bhai sells kites at Jamnagar. Six shopkeepers from nearby villages come to purchase kites from him. The number of kites he sold to these six shopkeepers are given below —

Shopkeeper	Number of Kites Sold
Chaman	250
Rani	300
Rukhsana	100
Jasmeet	450
Jetha Lal	250
Poonam Ben	700

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Prepare a pictograph using the symbol to represent 100 kites.

Answer the following questions:

a. How many symbols represent the kites that Rani purchased?

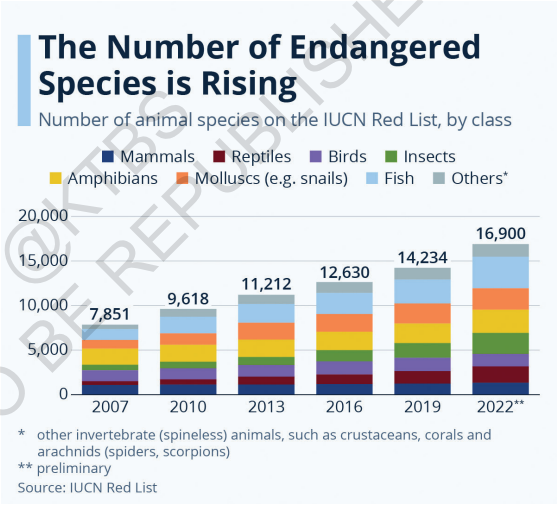
b. Who purchased the maximum number of kites?

c. Who purchased more kites, Jasmeet or Chaman?

d. Rukhsana says Poonam Ben purchased more than double the number of kites that Rani purchased. Is she correct? Why?

4.3 Bar Graphs

Have you seen graphs like this on TV or in a newspaper?



Source: https://www.statista.com/chart/17122/number-of-threatened-species-red-list/

Like pictographs, such **bar graphs** can help us to quickly understand and interpret information, such as the highest value, the comparison of values of different categories, etc. However, when the amount of data is large, presenting it by a pictograph is not only time consuming but at times difficult too. Let us see how data can be presented instead using a bar graph.

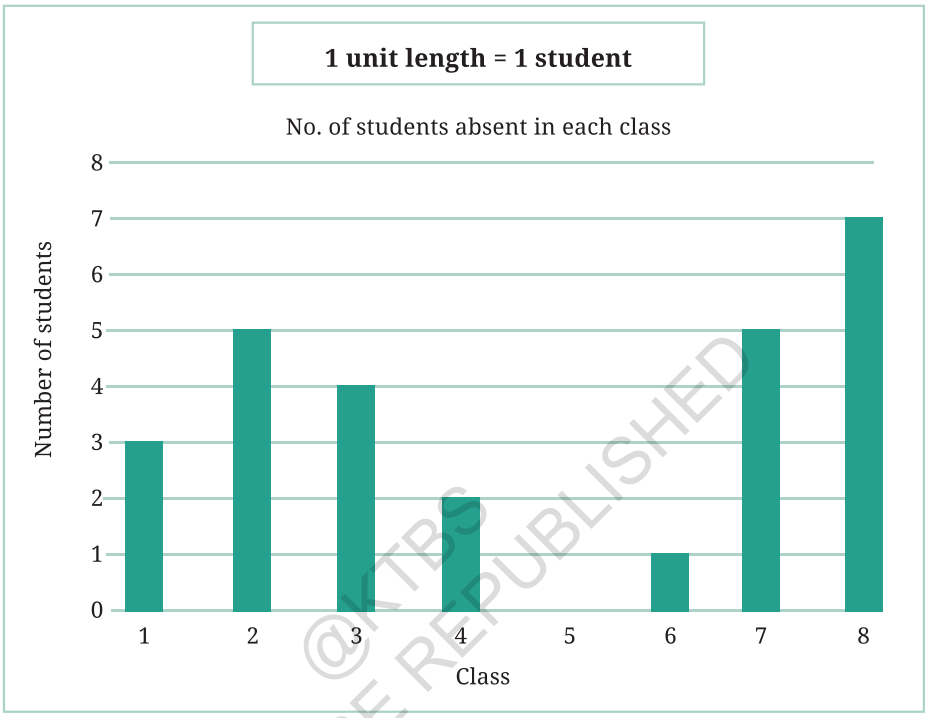
Let's take the data collected by Lakhanpal earlier, regarding the number of students absent on one day in each class_____

Class	1	2	3	4	5	6	7	8
No. of students	3	5	4	2	0	1	5	7

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He presented the same data using a bar graph:



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Teacher's Note

If the students have not noticed, please point out the equally spaced horizontal lines. Explain that this means that each pair of consecutive numbers on the left has the same gap.

Answer the following questions using the bar graph:

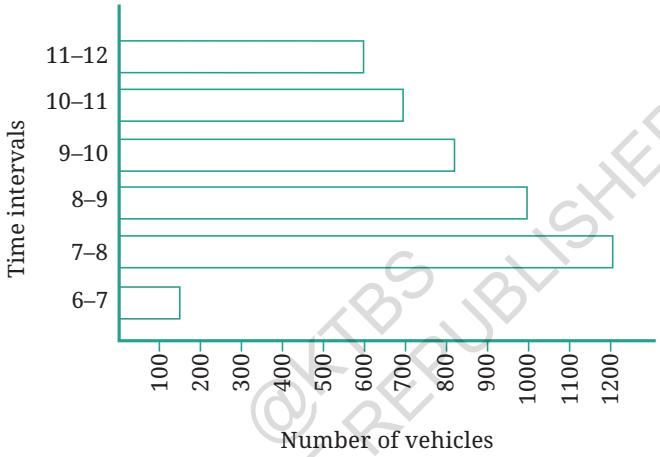
1. In Class 2, _____ students were absent that day.
2. In which class were the maximum number of students absent? _____
3. Which class had full attendance that day? _____

When making bar graphs, bars of uniform width can be drawn horizontally or vertically with equal spacing between them; then the

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length or height of each bar represents the given number. As we saw in pictographs, we can use a scale or key when the frequencies are larger.

Let us look at an example of vehicular traffic at a busy road crossing in Delhi, which was studied by the traffic police on a particular day. The number of vehicles passing through the crossing each hour from 6 a.m. to 12:00 noon is shown in the bar graph. One unit of length stands for 100 vehicles.



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We can see that the maximum traffic at the crossing is shown by the longest bar, i.e., for the time interval 7–8 a.m. The bar graph shows that 1200 vehicles passed through the crossing at that time. The second longest bar is for 8–9 a.m. During that time, 1000 vehicles passed through the crossing. Similarly, the minimum traffic is shown by the smallest bar, i.e., the bar for the time interval 6–7 a.m. During that time, only about 150 vehicles passed through the crossing. The second smallest bar is that for the time interval 11 a.m.–12 noon, when about 600 vehicles passed through the crossing.

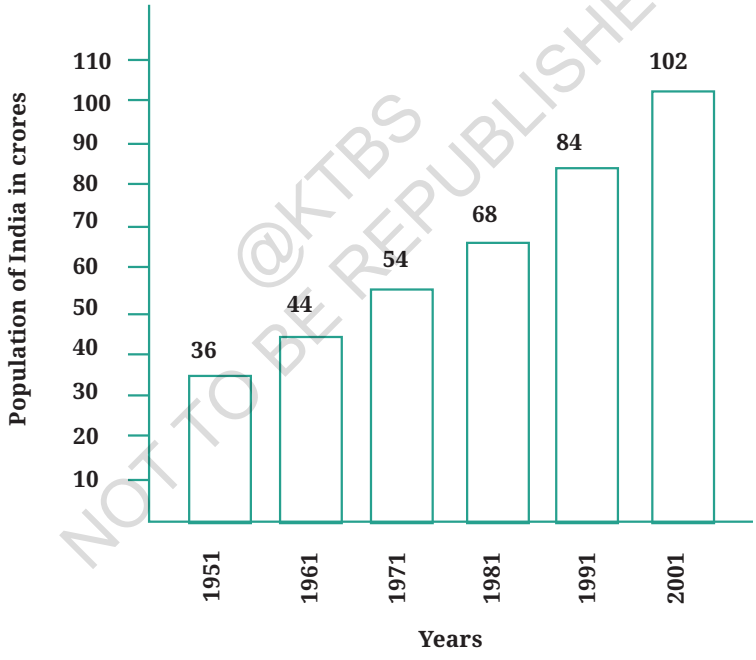
The total number of cars passing through the crossing during the two-hour interval 8.00–10.00 a.m. as shown by the bar graph is about 1000 + 800 = 1800 vehicles.

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Figure it Out

1. How many total cars passed through the crossing between 6 a.m. and noon?
2. Why do you think so little traffic occurred during the hour of 6–7 a.m., as compared to the other hours from 7 a.m.–noon?
3. Why do you think the traffic was the heaviest between 7–8 a.m.?
4. Why do you think the traffic was lesser and lesser each hour after 8 a.m. all the way until noon?

Example:



Population of India in crores

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This bar graph shows the population of India in each decade over a period of 50 years. The numbers are expressed in crores. If you were to take 1 unit length to represent one person, drawing the bars will

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be difficult! Therefore, we choose the scale so that 1 unit represents 10 crores. The bar graph for this choice is shown in the figure. So a bar of length 5 units represents 50 crores and of 8 units represents 80 crores.

- On the basis of this bar graph, what may be a few questions you may ask your friends?
- How much did the population of India increase over 50 years? How much did the population increase in each decade?

4.4 Drawing a Bar Graph

In a previous example, Shri Nilesh prepared a frequency table representing the sweet preferences of the students in his class. Let's try to prepare a bar graph to present his data —

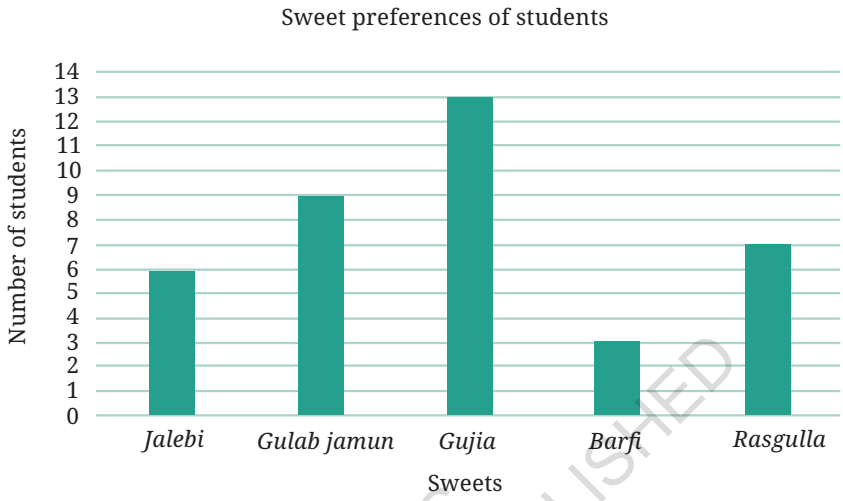
Sweet	No. of Students
Jalebi	6
Gulab jamun	9
Gujiya	13
Barfi	3
Rasgulla	7

1. First, we draw a horizontal line and a vertical line. On the horizontal line, we will write the name of each of the sweets, equally spaced, from which the bars will rise in accordance with their frequencies; and on the vertical line we will write the frequencies representing the number of students.
2. We must choose a scale. That means we must decide how many students will be represented by a unit length of a bar so that it fits nicely on our paper. Here, we will take 1 unit length to represent 1 student.
3. For *jalebi*, we therefore need to draw a bar having a height of 6 units (which is the frequency of the sweet *jalebi*), and similarly for the other sweets we have to draw bars as high as their frequencies.

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4. We, therefore, get a bar graph as shown below —



When the frequencies are larger and we cannot use the scale of 1 unit length = 1 number (frequency), we need to choose a different scale like we did in the case of pictographs.

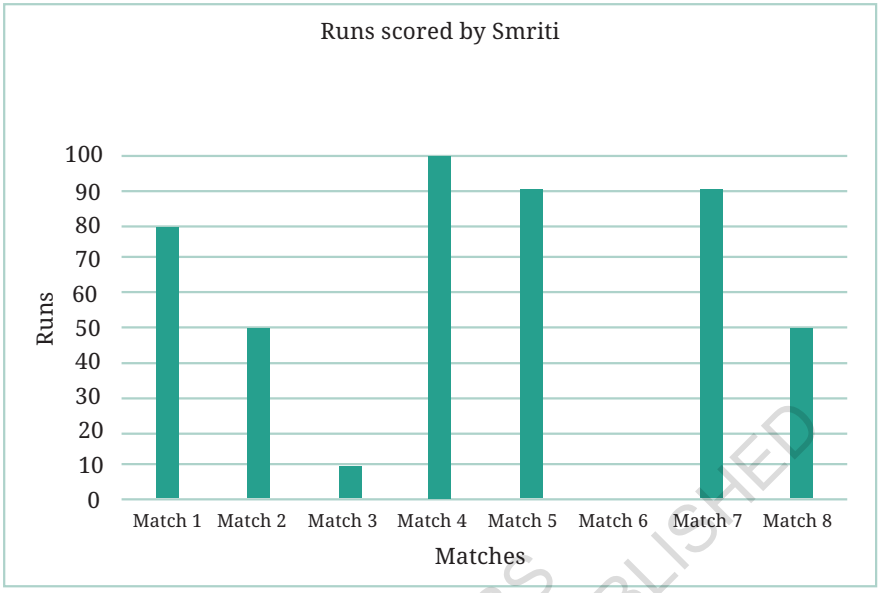
Example: The number of runs scored by Smriti in each of the 8 matches are given in the table below:

Match	Match 1	Match 2	Match 3	Match 4	Match 5	Match 6	Match 7	Match 8
Runs	80	50	10	100	90	0	90	50

In this example, the minimum score is 0 and the maximum score is 100. Using a scale of 1 unit length = l run would mean that we have to go all the way from 0 to 100 runs in steps of l. This would be unnecessarily tedious. Instead, let us use a scale where 1 unit length = 10 runs. We mark this scale on the vertical line and draw the bars according to the scores in each match. We get the following bar graph representing the above data.

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Example: The following table shows the monthly expenditure of Imran's family on various items:

Items	Expenditure (in ₹)
House rent	3000
Food	3400
Education	800
Electricity	400
Transport	600
Miscellaneous	1200

To represent this data in the form of a bar graph, here are the steps—

- Draw two perpendicular lines, one horizontal and one vertical.
- Along the horizontal line, mark the 'items' with equal spacing between them and mark the corresponding expenditures along the vertical line.

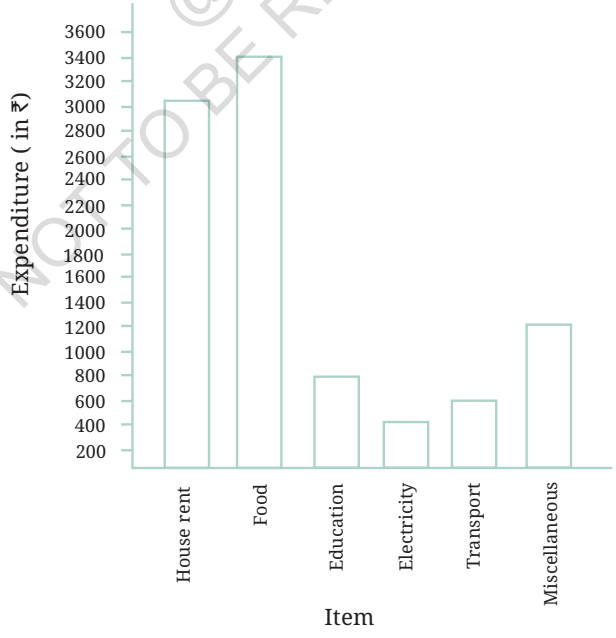
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- Take bars of the same width, keeping a uniform gap between them.
- Choose a suitable scale along the vertical line. Let, 1 unit length = ₹ 200, and then mark and write the corresponding values (₹ 200, ₹ 400, etc.) representing each unit length.

Finally, calculate the heights of the bars for various items as shown below —

House rent	3000 ÷ 200	15 units
Food	3400 ÷ 200	17 units
Education	800 ÷ 200	4 units
Electricity	400 ÷ 200	2 units
Transport	600 ÷ 200	3 units
Miscellaneous	1200 ÷ 200	6 units

Here is the bar graph that we obtain based on the above steps:



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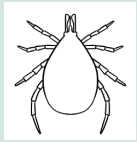
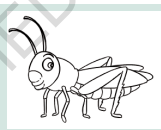
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Use the bar graph to answer the following questions:

1. On which item does Imran's family spend the most and the second most?
2. Is the cost of electricity about one-half the cost of education?
3. Is the cost of education less than one-fourth the cost of food?

Figure it Out

1. Samantha visited a tea garden, and collected data of the insects and critters she saw there. Here is the data she collected:

				
Mites	Caterpillars	Beetles	Butterflies	Grasshoppers
6	10	5	3	2

Help her prepare a bar graph representing this data.

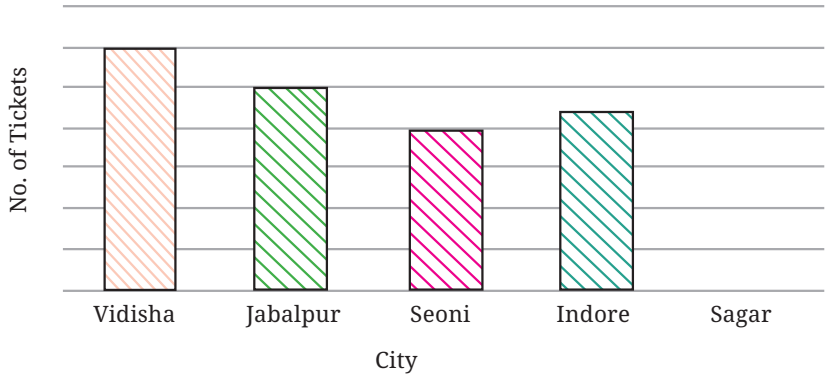
2. Pooja collected data on the number of tickets sold at the Bhopal railway station for a few different cities of Madhya Pradesh over a two-hour period.

City	Vidisha	Jabalpur	Seoni	Indore	Sagar
Number of tickets	24	20	16	28	16

She used this data and prepared a bar graph on the board to discuss the data with her students, but someone erased a portion of the graph.

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a. Write the number of tickets sold for Vidisha above the bar.

b. Write the number of tickets sold for Jabalpur above the bar.

c. The bar for Vidisha is 6 unit lengths and the bar for Jabalpur is 5 unit lengths. What is the scale for this graph?

d. Draw the correct bar for Sagar.

e. Add the scale of the bar graph by placing the correct numbers on the vertical axis.

f. Are the bars for Seoni and Indore correct in this graph? If not, draw the correct bar(s).

3. Chinu listed the various means of transport that passed across the road in front of his house from 9 a.m. to 10 a.m.:

bike	car	bike	bus	bike	bike
bike	auto rickshaw	bicycle	bullock cart	bicycle	auto rickshaw
car	scooter	car	auto rickshaw	bicycle	bike
car	auto rickshaw	bike	scooter	bike	car
bicycle	scooter	bicycle	scooter	bike	bus
auto rickshaw	auto rickshaw	bike	bicycle	bus	bike
bicycle	scooter	bus	scooter	auto rickshaw	bike
scooter	bicycle	bike	bullock cart	auto rickshaw	scooter
car	scooter				

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a. Prepare a frequency distribution table for the data.

b. Which means of transport was used the most?

c. If you were there to collect this data, how could you do it? Write the steps or process.

4. Roll a die 30 times and record the number you obtain each time. Prepare a frequency distribution table using tally marks. Find the number that appeared:

a. The minimum number of times.

b. The maximum number of times.

c. Find numbers that appeared an equal number of times.

5. Faiz prepared a frequency distribution table of data on the number of wickets taken by Jaspreet Bumrah in his last 30 matches:

Wickets Taken	Number of Matches
0	2
1	4
2	6
3	8
4	3
5	5
6	1
7	1

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a. What information is this table giving?

b. What may be the title of this table?

c. What caught your attention in this table?

d. In how many matches has Bumrah taken 4 wickets?

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e. Mayank says, "If we want to know the total number of wickets he has taken in his last 30 matches, we have to add the numbers 0, 1, 2, 3 ..., up to 7." Can Mayank get the total number of wickets taken in this way? Why?

f. How would you correctly figure out the total number of wickets taken by Bumrah in his last 30 matches, using this table?

6. The following pictograph shows the number of tractors in five different villages.

Villages	Number of Tractors (= 1 Tractor)
Village A	
Village B	
Village C	
Village D	
Village E	

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Observe the pictograph and answer the following questions—

a. Which village has the smallest number of tractors?

b. Which village has the most tractors?

c. How many more tractors does Village C have than Village B?

d. Komal says, "Village D has half the number of tractors as Village E." Is she right?

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7. The number of girl students in each class of a school is depicted by the pictograph:

Classes	Number of Girl Students ( = 4 Girls)
1	
2	
3	
4	
5	
6	
7	
8	

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Observe this pictograph and answer the following questions:

a. Which class has the least number of girl students?

b. What is the difference between the number of girls in Classs 5 and 6?

c. If two more girls were admitted in Class 2, how would the graph change?

d. How many girls are there in Class 7?

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8. Mudhol Hounds (a type of breed of Indian dogs) are largely found in North Karnataka's Bagalkote and Vijaypura districts. The government took an initiative to protect this breed by providing support to those who adopted these dogs. Due to this initiative, the number of these dogs increased. The number of Mudhol dogs in six villages of Karnataka are as follows—

   Village A : 18, Village B : 36, Village C : 12, Village D : 48, Village E : 18, Village F : 24

   Prepare a pictograph and answer the following questions:

   a. What will be a useful scale or key to draw this pictograph?

   b. How many symbols will you use to represent the dogs in Village B?

   c. Kamini said that the number of these dogs in Village B and Village D together will be more than the number of these dogs in the other 4 villages. Is she right? Give reasons for your response.

9. A survey of 120 school students was conducted to find out which activity they preferred to do in their free time:

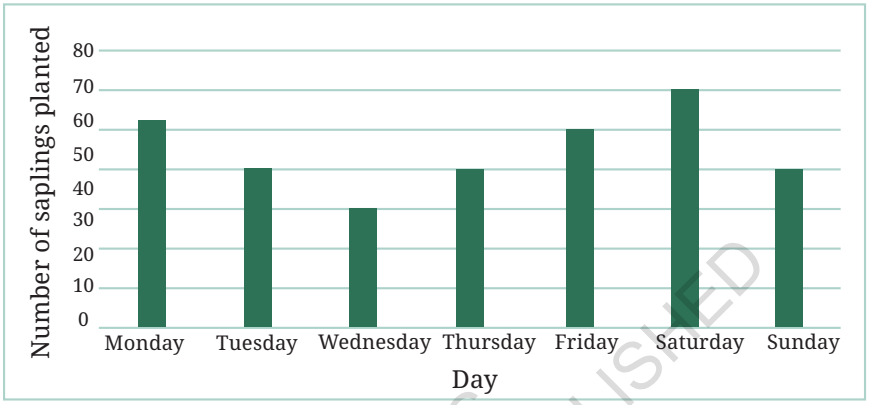
Preferred Activity	Number of Students
Playing	45
Reading story books	30
Watching TV	20
Listening to music	10
Painting	15

   Draw a bar graph to illustrate the above data taking the scale of 1 unit length = 5 students. Which activity is preferred by most students other than playing?

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10. Students and teachers of a primary school decided to plant tree saplings in the school campus and in the surrounding village during the first week of July. Details of the saplings they planted are as follows —



a. The total number of saplings planted on Wednesday and Thursday is _____.

b. The total number of saplings planted during the whole week is _____.

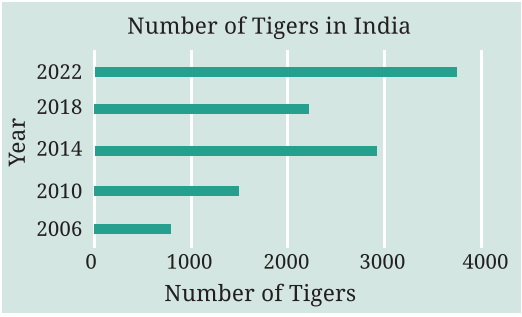
c. The greatest number of saplings were planted on _____ and the least number of saplings were planted on _____. Why do you think that is the case? Why were more saplings planted on certain days of the week and less on others? Can you think of possible explanations or reasons? How could you try and figure out whether your explanations are correct?

11. The number of tigers in India went down drastically between 1900 and 1970. Project Tiger was launched in 1973 to track and protect the tigers in India. Starting in 2006, the exact number of tigers in India was tracked. Shagufta and Divya looked up information about the number of tigers in India between 2006 and 2022 in four-year intervals. They prepared a frequency table for this data and a bar graph to present this data, but there are a few mistakes in the graph. Can you find those mistakes and fix them?

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Year	Number of Tigers (approx.)
2006	1400
2010	1700
2014	2200
2018	3000
2022	3700



- Like pictographs, bar graphs give a nice visual way to represent data. They represent data through equally-spaced bars, each of equal width, where the lengths or heights give frequencies of the different categories.
- Each category is represented by a bar where the length or height depicts the corresponding frequency (for example, cost) or quantity (for example, runs).
- The bars have uniform spaces between them to indicate that they are free standing and represent equal categories.
- The bars help in interpreting data much faster than a frequency table. By reading a bar graph, we can compare frequencies of different categories at a glance.
- We must decide the scale (for example, 1 unit length = 1 student or 1 unit length = ₹ 200) for a bar graph on the basis of the data including the minimum and maximum frequencies, so that the resulting bar graph fits nicely and looks visually appealing on the paper or poster we are preparing. The markings of the unit lengths as per the scale must start from zero.

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Teacher's Note

The main focus of this chapter is to learn how to handle data to find answers to specific questions or inquiries, to test hypotheses or to take specific decisions. This should be kept in mind when providing practice opportunities to collect, organise and analyse data.

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4.5 Artistic and Aesthetic Considerations

In addition to the steps described in previous sections, there are also some other more artistic and aesthetic aspects one can consider when preparing visual presentations of data to make them more interesting and effective. First, when making a visual presentation of data such as a pictograph or bar graph, it is important to make it fit in the intended space; this can be controlled, for example, by choosing the scale appropriately, as we have seen earlier. It is also desirable to make the data presentation visually appealing and easy-to-understand, so that the intended audience appreciates the information being conveyed.

Let us consider an example. Here is a table naming the tallest mountain on each continent, along with the height of each mountain in meters.

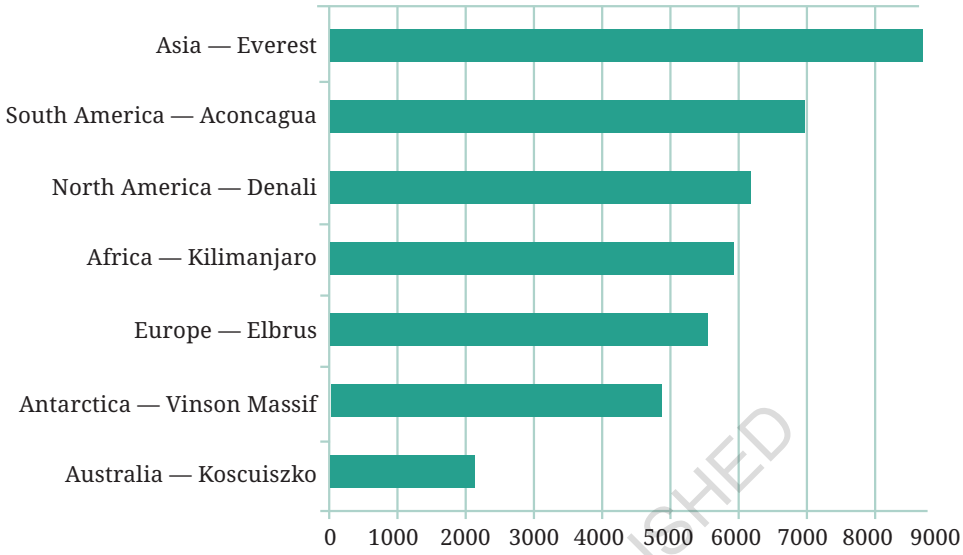
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Continent	Asia	South America	North America	Africa	Europe	Antarctica	Australia
Tallest Mountain	Everest	Aconcagua	Denali	Kiliman-jaro	Elbrus	Vinson Massif	Koscuiszko
Height	8848m	6962m	6194m	5895m	5642m	4892m	2228m

How much taller is Mount Everest than Mount Koscuiszko? Are Mount Denali and Mount Kilimanjaro very different in height? This is not so easy to quickly discern from a large table of numbers.

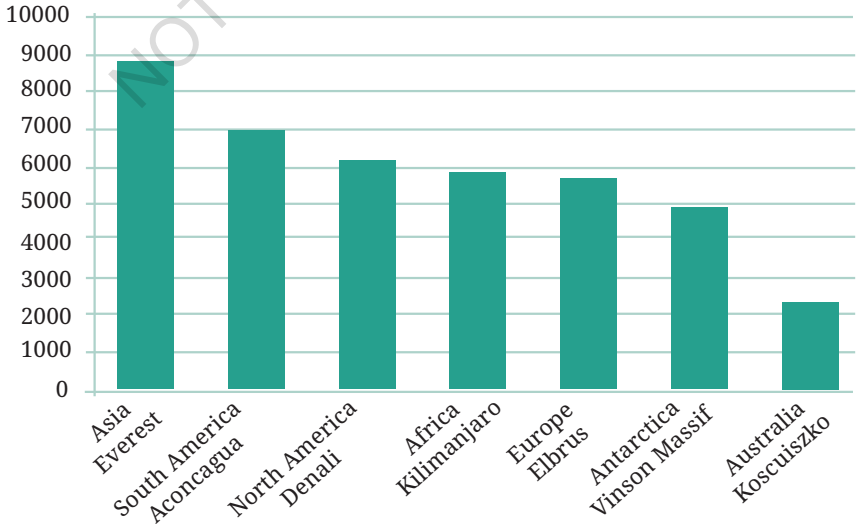
As we have seen earlier, we can convert the table of numbers into a bar graph, as shown on the right. Here, each value is drawn as a horizontal box. These are longer or shorter depending on the number they represent. This makes it easier to compare the heights of all these mountains at a glance.

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However, since the boxes represent heights, it is better and more visually appealing to rotate the picture, so that the boxes grow upward, vertically from the ground like mountains. A bar graph with vertical bars is also called a column graph. Columns are the pillars you find in a building that hold up the roof.

Below is a column graph for our table of tallest mountains. From this column graph, it becomes easier to compare and visualise the heights of the mountains.



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In general, it is more intuitive, suggestive and visually appealing to represent heights, that are measured upwards from the ground, using bar graphs that have vertical bars or columns. Similarly, lengths that are parallel to the ground (for example, distances between location on Earth) are usually best represented using bar graphs with horizontal arcs.

Figure it Out

1. If you wanted to visually represent the data of the heights of the tallest persons in each class in your school, would you use a graph with vertical bars or horizontal bars? Why?
2. If you were making a table of the longest rivers on each continent and their lengths, would you prefer to use a bar graph with vertical bars or with horizontal bars? Why? Try finding out this information, and then make the corresponding table and bar graph! Which continents have the longest rivers?

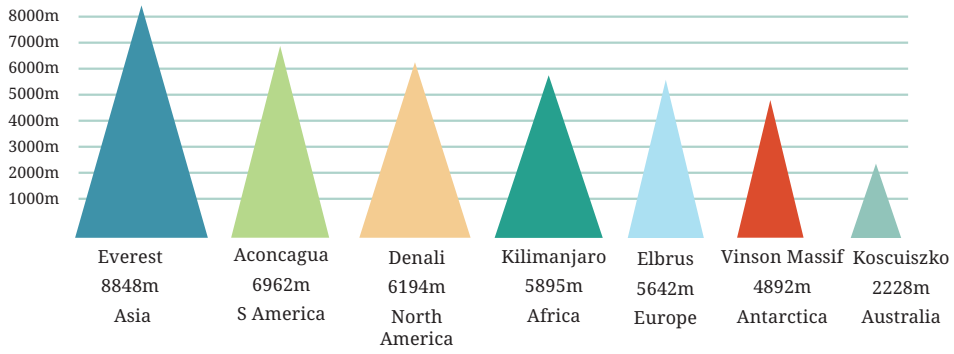
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Infographics

When data visualisations such as bar graphs are further beautified with more extensive artistic and visual imagery, they are called **information graphics** or **infographics** for short. The aim of infographics is to make use of attention-attracting and engaging visuals to communicate information even more clearly and quickly, in a visually pleasing way.

As an example of how infographics can be used to communicate data even more suggestively, let us go back to the table above listing the tallest mountain on each continent. We drew a bar graph with vertical bars (columns) rather than horizontal bars, to be more indicative of mountains. But instead of rectangles, we could use triangles, which look a bit more like mountains. And, we can add a splash of colour as well. Here is the result.

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While this infographic might look more appealing and suggestive at first glance, it does have some issues. The goal of our bar graph earlier was to represent the heights of various mountains — using bars of the appropriate heights but the same widths. The purpose of using the same widths was to make it clear that we are only comparing heights. However, in this infographic, the taller triangles are also wider! Are taller mountains always wider? The infographic is implying additional information that may be misleading and may or may not be correct. Sometimes going for more appealing pictures can also accidentally mislead.

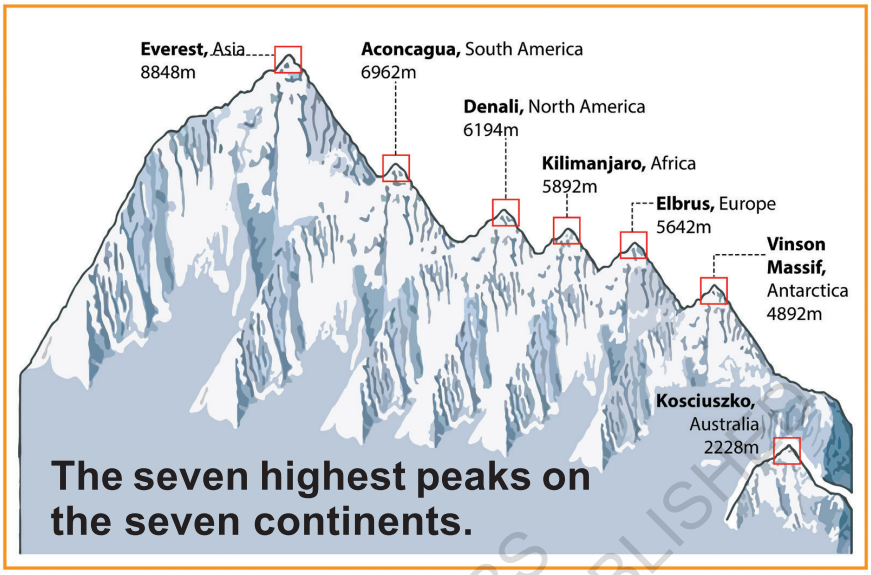
Taking this idea further, and to make the picture even more visually stimulating and suggestive, we can further change the shapes of the mountains to make them look even more like mountains, and add other details, while attempting to preserve the heights. For example, we can create an imaginary mountain range that contains all these mountains.

Is the infographic below better than the column graph with rectangular columns of equal width? The mountains look more realistic, but is the picture accurate?

For example, Everest appears to be twice as tall as Elbrus.

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What is 5642×2?

While preparing visually-appealing presentations of data, we also need to be careful that the pictures we draw do not mislead us about the facts. In general, it is important to be careful when making or reading infographics, so that we do not mislead our intended audiences and we, ourselves, are not misled.

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SUMMARY

- Facts, numbers, measures, observations and other descriptions of things that convey information about those things is called **data**.
- Data can be organised in a tabular form using tally marks for easy analysis and interpretation.
- **Frequencies** are the counts of the occurrences of values, measures or observations.

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- **Pictographs** represent data in the form of pictures, or objects or parts of objects. Each picture represents a frequency which can be 1 or more than 1 — this is called the **scale** and it must be specified.
- **Bar graphs** have bars of uniform width; the length or height that indicates the total frequency of occurrence. The **scale** that is used to convert length or height to frequency again, must be specified.
- Choosing the appropriate scale for a pictograph or bar graph is important to accurately and effectively convey the desired information or data and to also make it visually appealing.
- Other aspects of a graph also contribute to its effectiveness and visual appeal such as how colours are used, what accompanying pictures are drawn, and whether the bars are horizontal or vertical. These aspects correspond to the artistic and aesthetic side of data handling and presentation.
- However, making visual representations of data 'fancy' can also sometimes be misleading.
- By reading pictographs and bar graphs accurately, we can quickly understand and make inferences about the data presented.

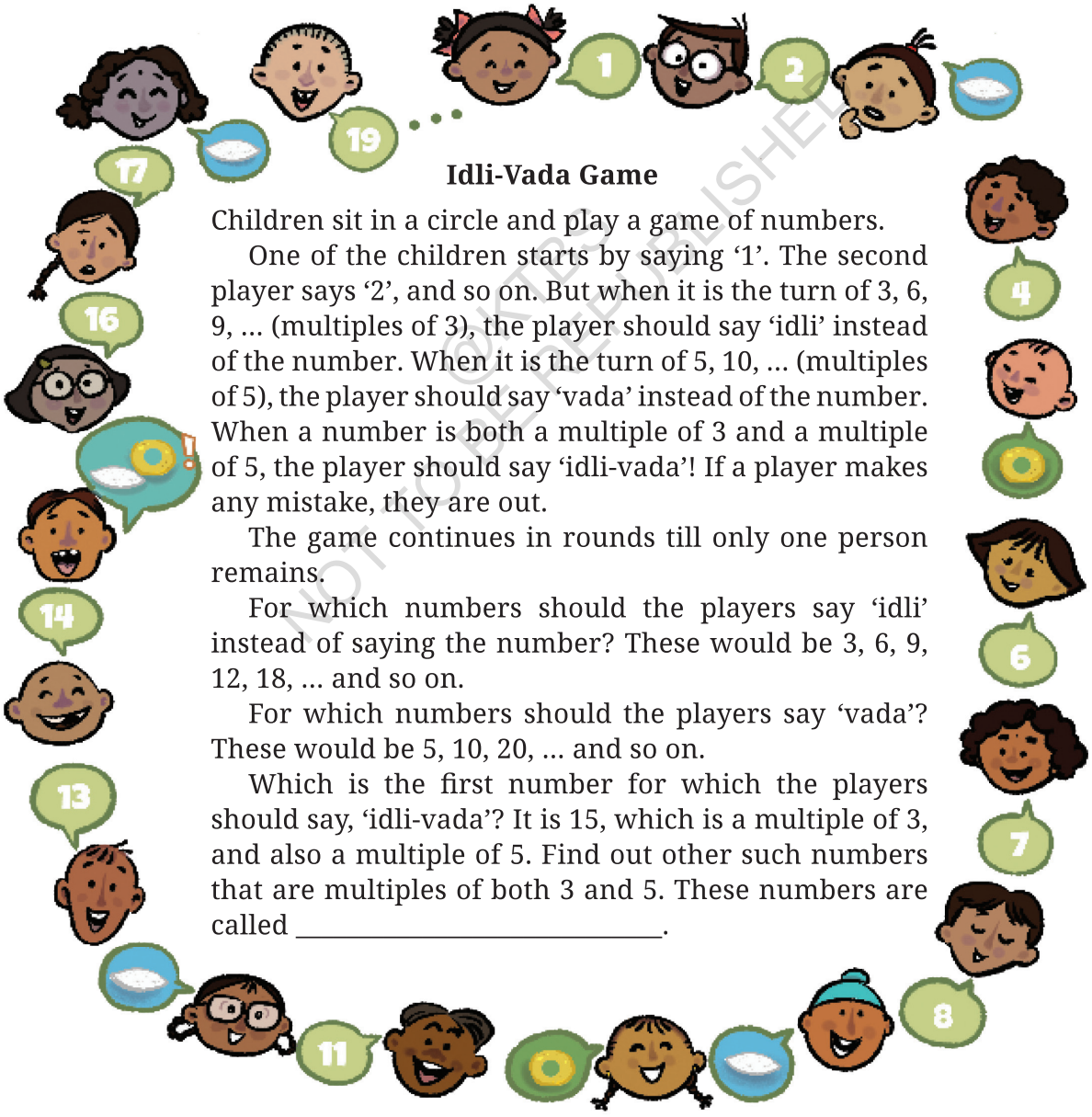
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5 PRIME TIME

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5.1 Common Multiples and Common Factors



Idli-Vada Game

Children sit in a circle and play a game of numbers.

One of the children starts by saying '1'. The second player says '2', and so on. But when it is the turn of 3, 6, 9, ... (multiples of 3), the player should say 'idli' instead of the number. When it is the turn of 5, 10, ... (multiples of 5), the player should say 'vada' instead of the number. When a number is both a multiple of 3 and a multiple of 5, the player should say 'idli-vada'! If a player makes any mistake, they are out.

The game continues in rounds till only one person remains.

For which numbers should the players say 'idli' instead of saying the number? These would be 3, 6, 9, 12, 18, ... and so on.

For which numbers should the players say 'vada'? These would be 5, 10, 20, ... and so on.

Which is the first number for which the players should say, 'idli-vada'? It is 15, which is a multiple of 3, and also a multiple of 5. Find out other such numbers that are multiples of both 3 and 5. These numbers are called _____.

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Figure it Out

1. At what number is 'idli-vada' said for the 10th time?
2. If the game is played for the numbers 1 to 90, find out:
   a. How many times would the children say 'idli' (including the times they say 'idli-vada')?
   b. How many times would the children say 'vada' (including the times they say 'idli-vada')?
   c. How many times would the children say 'idli-vada'?
3. What if the game was played till 900? How would your answers change?
4. Is this figure somehow related to the 'idli-vada' game?

   Hint: Imagine playing the game till 30. Draw the figure if the game is played till 60.

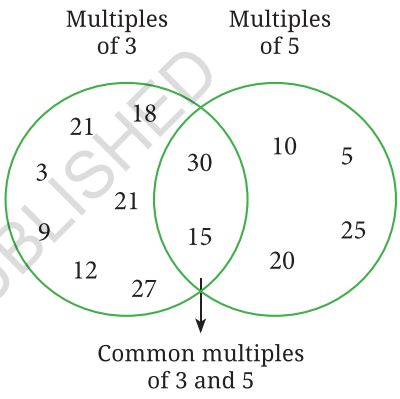
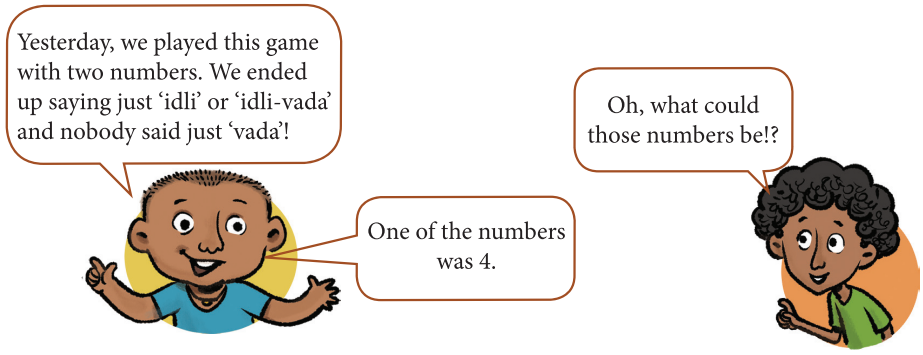


Fig. 5.1

Let us now play the 'idli-vada' game with different pairs of numbers:

a. 2 and 5,
b. 3 and 7,
c. 4 and 6.

We will say 'idli' for multiples of the smaller number, 'vada' for multiples of the larger number and 'idli-vada' for common multiples. Draw a figure similar to Fig. 5.1 if the game is played up to 60.



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Which of the following could be the other number:

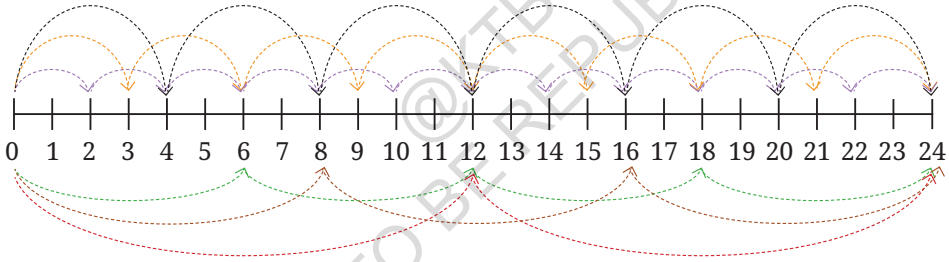
2, 3, 5, 8, 10?

Jump Jackpot

Jumpy and Grumpy play a game.

- Grumpy places a treasure on some number. For example, he may place it on 24.
- Jumpy chooses a jump size. If he chooses 4, then he has to jump only on multiples of 4, starting at 0.
- Jumpy gets the treasure if he lands on the number where Grumpy placed it.

Which jump sizes will get Jumpy to land on 24?
If he chooses 4: Jumpy lands on 4 → 8 → 12 → 16 → 20 → 24 → 28 → ...
Other successful jump sizes are 2, 3, 6, 8 and 12.



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What about the jump of sizes 1 and 24? Yes, they also will land on 24.

The numbers 1, 2, 3, 4, 6, 8, 12, 24 all divide 24 exactly. Recall that such numbers are called **factors** or **divisors** of 24.

Grumpy increases the level of the game. Two treasures are kept on two different numbers. Jumpy has to choose a jump size and stick to it. Jumpy gets the treasures only if he lands on both the numbers with the chosen jump size. As before, Jumpy starts at 0.

Grumpy has kept the treasures on 14 and 36. And, Jumpy chooses a jump size of 7.

Will Jumpy land on both the treasures? Starting from 0, he jumps to 7 → 14 → 21 → 28 → 35 → 42 ... We see that he landed on 14 but

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did not land on 36, so he does not get the treasure. What jump size should he have chosen?

The factors of 14 are: 1, 2, 7, 14. So, these jump sizes will land on 14.

The factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36. These jump sizes will land on 36.

So, the jump sizes of 1 or 2 will land on both 14 and 36. Notice that 1 and 2 are the common factors of 14 and 36.

The jump sizes using which both the treasures can be reached are the **common factors** of the two numbers where the treasures are placed.

- What jump size can reach both 15 and 30? There are multiple jump sizes possible. Try to find them all.

- Look at the table below. What do you notice?

31	(32)	33	34	35	(36)	37	38	39	(40)
41	42	43	(44)	45	46	47	(48)	49	50
51	(52)	53	54	55	(56)	57	58	59	(60)
61	62	63	(64)	65	66	67	(68)	69	70

In the table,

1. Is there anything common among the shaded numbers?
2. Is there anything common among the circled numbers?
3. Which numbers are both shaded and circled? What are these numbers called?

Math Talk

- **Figure it Out**

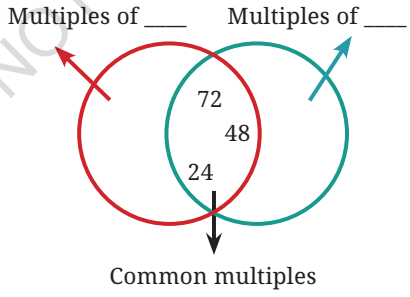
1. Find all multiples of 40 that lie between 310 and 410.

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2. Who am I?
   a. I am a number less than 40. One of my factors is 7. The sum of my digits is 8.
   b. I am a number less than 100. Two of my factors are 3 and 5. One of my digits is 1 more than the other.
3. A number for which the sum of all its factors is equal to twice the number is called a **perfect number**. The number 28 is a perfect number. Its factors are 1, 2, 4, 7, 14 and 28. Their sum is 56 which is twice 28. Find a perfect number between 1 and 10.
4. Find the common factors of:
   a. 20 and 28 b. 35 and 50
   c. 4, 8 and 12 d. 5, 15 and 25
5. Find any three numbers that are multiples of 25 but not multiples of 50.
6. Anshu and his friends play the 'idli-vada' game with two numbers, which are both smaller than 10. The first time anybody says 'idli-vada' is after the number 50. What could the two numbers be which are assigned 'idli' and 'vada'?
7. In the treasure hunting game, Grumpy has kept treasures on 28 and 70. What jump sizes will land on both the numbers?
8. In the diagram below, Guna has erased all the numbers except the common multiples. Find out what those numbers could be and fill in the missing numbers in the empty regions.

Math Talk



9. Find the smallest number that is a multiple of all the numbers from 1 to 10, except for 7.
10. Find the smallest number that is a multiple of all the numbers from 1 to 10.

Try This

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5.2 Prime Numbers

Guna and Anshu want to pack figs (*anjeer*) that grow in their farm. Guna wants to put 12 figs in each box and Anshu wants to put 7 figs in each box.

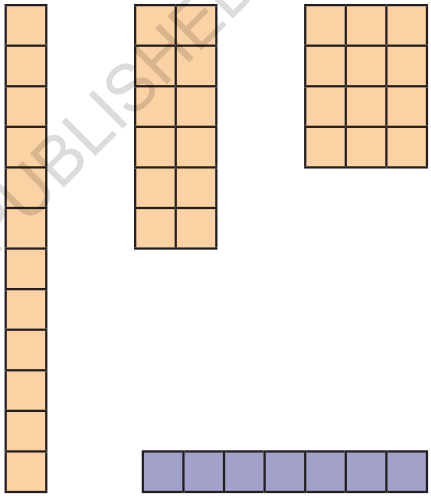
How many arrangements are possible?

Think and find out the different ways how—

1. Guna can arrange 12 figs in a rectangular manner.
2. Anshu can arrange 7 figs in a rectangular manner.

Guna has listed out these possibilities.

Observe the number of rows and columns in each of the arrangements. How are they related to 12?



In the second arrangement, for example, 12 figs are arranged in two columns of 6 each or $12 = 2 \times 6$.

Anshu could make only one arrangement: 7×1 or 1×7. There are no other rectangular arrangements possible.

In each of Guna's arrangements, multiplying the number of rows by the number of columns gives the number 12. So, the number of rows or columns are factors of 12.

We saw that the number 12 can be arranged in a rectangle in more than one way as 12 has more than two factors. The number 7 can be arranged in only one way, as it has only two factors—1 and 7.

Numbers that have only two factors are called **prime numbers** or *primes*. Here are the first few primes—2, 3, 5, 7, 11, 13, 17, 19. Notice that the factors of a prime number are 1 and the number itself.

What about numbers that have more than two factors? They are called **composite numbers**. The first few composite numbers are—4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.

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What about 1, which has only one factor? The number 1 is neither a prime nor a composite number.

How many prime numbers are there from 21 to 30? How many composite numbers are there from 21 to 30?

Can we list all the prime numbers from 1 to 100?

Here is an interesting way to find prime numbers. Just follow the steps given below and see what happens.

Step 1: Cross out 1 because it is neither prime nor composite.

Step 2: Circle 2, and then cross out all multiples of 2 after that, i.e., 4, 6, 8, and so on.

Step 3: You will find that the next uncrossed number is 3. Circle 3 and then cross out all the multiples of 3 after that, i.e., 6, 9, 12, and so on.

Step 4: The next uncrossed number is 5. Circle 5 and then cross out all the multiples of 5 after that, i.e., 10, 15, 20, and so on.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 5: Continue this process till all the numbers in the list are either circled or crossed out.

All the circled numbers are prime numbers. All the crossed out numbers, other than 1, are composite numbers. This method is called the Sieve of Eratosthenes.

This procedure can be carried on for numbers greater than 100 also. Eratosthenes was a Greek mathematician who lived around 2200 years ago and developed this method of listing primes.



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Guna and Anshu started wondering how this simple method is able to find prime numbers! Think how this method works. Read the steps given above again and see what happens after each step is carried out.

Figure it Out

1. We see that 2 is a prime and also an even number. Is there any other even prime?
2. Look at the list of primes till 100. What is the smallest difference between two successive primes? What is the largest difference?
3. Are there an equal number of primes occurring in every row in the table on the previous page? Which decades have the least number of primes? Which have the most number of primes?

Primes through the Ages

Prime numbers are the building blocks of all whole numbers. Starting from the time of the Greek civilisation (more than 2000 years ago) to this day, mathematicians are still struggling to uncover their secrets!

Food for thought: is there a largest prime number? Or does the list of prime numbers go on without an end? A mathematician named Euclid found the answer and so will you in a later class!

Fun fact: The largest prime number that anyone has 'written down' is so large that it would take around 6500 pages to write it! So they could only write it on a computer!

4. Which of the following numbers are prime: 23, 51, 37, 26?
5. Write three pairs of prime numbers less than 20 whose sum is a multiple of 5.
6. The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers up to 100.
7. Find seven consecutive composite numbers between 1 and 100.
8. **Twin primes** are pairs of primes having a difference of 2. For example, 3 and 5 are twin primes. So are 17 and 19. Find the other twin primes between 1 and 100.

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9. Identify whether each statement is true or false. Explain.
   a. There is no prime number whose units digit is 4.
   b. A product of primes can also be prime.
   c. Prime numbers do not have any factors.
   d. All even numbers are composite numbers.
   e. 2 is a prime and so is the next number, 3. For every other prime, the next number is composite.
10. Which of the following numbers is the product of exactly three distinct prime numbers: 45, 60, 91, 105, 330?
11. How many three-digit prime numbers can you make using each of 2, 4 and 5 once?
12. Observe that 3 is a prime number, and $2 \times 3 + 1 = 7$ is also a prime. Are there other primes for which doubling and adding 1 gives another prime? Find at least five such examples.

5.3 Co-prime Numbers for Safekeeping Treasures

Which pairs are safe?

Let us go back to the treasure finding game. This time, treasures are kept on two numbers. Jumpy gets the treasures only if he is able to reach both the numbers with the same jump size. There is also a new rule—a jump size of 1 is not allowed.

Where should Grumpy place the treasures so that Jumpy cannot reach both the treasures?

Will placing the treasure on 12 and 26 work? No! If the jump size is chosen to be 2, then Jumpy will reach both 12 and 26.

What about 4 and 9? Jumpy cannot reach both using any jump size other than 1. So, Grumpy knows that the pair 4 and 9 is safe.

Check if these pairs are safe:

a. 15 and 39 b. 4 and 15
c. 18 and 29 d. 20 and 55

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What is special about safe pairs? They don't have any common factor other than 1. Two numbers are said to be **co-prime** to each other if they have no common factor other than 1.

Example: As 15 and 39 have 3 as a common factor, they are not co-prime. But 4 and 9 are co-prime.

Which of the following pairs of numbers are co-prime?

a. 18 and 35 b. 15 and 37 c. 30 and 415
d. 17 and 69 e. 81 and 18

While playing the '*idli-vada*' game with different number pairs, Anshu observed something interesting!

Math Talk

1. Sometimes the first common multiple was the same as the product of the two numbers.
2. At other times the first common multiple was less than the product of the two numbers.

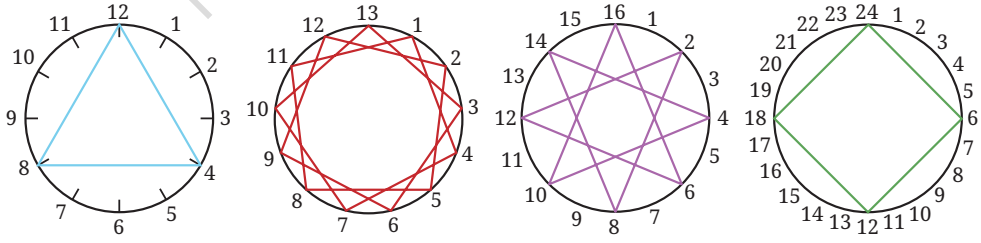
Find examples for each of the above. How is it related to the number pair being co-prime?

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Co-prime art

Observe the following thread art. The first diagram has 12 pegs and the thread is tied to every fourth peg (we say that the thread-gap is 4). The second diagram has 13 pegs and the thread-gap is 3. What about the other diagrams? Observe these pictures, share and discuss your findings in class.

Math Talk



In some diagrams, the thread is tied to every peg. In some, it is not. Is it related to the two numbers (the number of pegs and the thread-gap) being co-prime?

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Make such pictures for the following:

a. 15 pegs, thread-gap of 10
b. 10 pegs, thread-gap of 7
c. 14 pegs, thread-gap of 6
d. 8 pegs, thread-gap of 3

5.4 Prime Factorisation

Checking if two numbers are co-prime

Teacher: Are 56 and 63 co-prime?

Anshu and Guna: If they have a common factor other than 1, then they are not co-prime. Let us check.

Anshu: I can write $56 = 14 \times 4$ and $63 = 21 \times 3$. So, 14 and 4 are factors, of 56. Further, 21 and 3 are factors of 63. So, there are no common factors. The numbers are co-prime.

Guna: Hold on. I can also write $56 = 7 \times 8$ and $63 = 9 \times 7$. We see that 7 is a factor of both numbers, so, they are not co-prime.

Clearly Guna is right, as 7 is a common factor.

But where did Anshu go wrong?

Writing $56 = 14 \times 4$ tells us that 14 and 4 are both factors of 56, but it does not tell all the factors of 56. The same holds for the factors of 63.

Try another example: 80 and 63. There are many ways to factorise both numbers.

$80 = 40 \times 2 = 20 \times 4 = 10 \times 8 = 16 \times 5 = ???$

$63 = 9 \times 7 = 3 \times 21 = ???$

We have written '???' to say that there may be more ways to factorise these numbers. But if we take any of the given factorisations, for example, $80 = 16 \times 5$ and $63 = 9 \times 7$, then there are no common factors. Can we conclude that 80 and 63 are co-prime? As Anshu's mistake above shows, we cannot conclude that as there may be other ways to factorise the numbers.

What this means is that we need a more systematic approach to check if two numbers are co-prime.

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Prime factorisation

Take a number such as 56. It is composite, as we saw that it can be written as **56 = 4 × 14** . So, both 4 and 14 are factors of 56. Now take one of these, say 14. It is also composite and can be written as $14 = 2 \times 7$. Therefore, **56 = 4 × 2 × 7**. Now, 4 is composite and can be written as $4 = 2 \times 2$. Therefore, **56 = 2 × 2 × 2 × 7**. All the factors appearing here, 2 and 7, are prime numbers. So, we cannot divide them further.

In conclusion, we have written 56 as a product of prime numbers. This is called **prime factorisation** of 56. The individual factors are called *prime factors*. For example, the prime factors of 56 are 2 and 7.

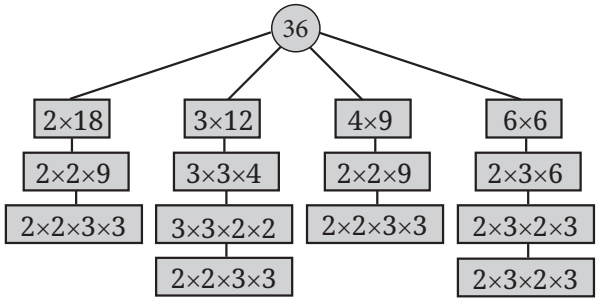
Every number greater than 1 has a prime factorisation. The idea is the same: keep breaking the composite numbers into factors till only primes are left.

The number 1 does not have any prime factorisation. It is not divisible by any prime number.

What is the prime factorisation of a prime number like 7? It is just 7 (we cannot break it down any further).

Let us see a few more examples.

By going through different ways of breaking down the number, we wrote 63 as $3 \times 3 \times 7$ and as $3 \times 7 \times 3$. Are they different? Not really! The same prime numbers 3 and 7 occur in both cases. Further, 3 appears two times in both and 7 appears once.



Here, you see four different ways to get prime factorisation of 36. Observe that in all four cases, we get two 2s and two 3s.

Multiply back to see that you get 36 in all four cases.

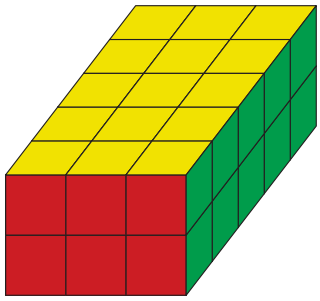
For any number, it is a remarkable fact that there is only one prime factorisation, except that the prime factors may come in different

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orders. As we explain below, the order is not important. However, as we saw in these examples, there are many ways to arrive at the prime factorisation!

Does the order matter?



Using this diagram,
can you explain why **30 = 2 × 3 × 5,** no matter which way you multiply 2, 3, and 5?

When multiplying numbers, we can do so in any order. The end result is the same. That is why, when two 2s and two 3s are multiplied in any order, we get 36. In a later class, we shall study this under the names of **commutativity and associativity of multiplication**.

Thus, the order does not matter. Usually we write the prime numbers in increasing order. For example, 225 = 3 × 3 × 5 × 5 or 30 = 2 × 3 × 5.

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Prime factorisation of a product of two numbers

When we find the prime factorisation of a number, we first write it as a product of two factors. For example, 72 = 12 × 6. Then, we find the prime factorisation of each of the factors. In the above example, 12 = 2 × 2 × 3 and 6 = 2 × 3. Now, can you say what the prime factorisation of 72 is?

The prime factorisation of the original number is obtained by putting these together.

$$72 = 2 \times 2 \times 3 \times 2 \times 3$$

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We can also write this as $2 \times 2 \times 2 \times 3 \times 3$. Multiply and check that you get 72 back!

Observe how many times each prime factor occurs in the factorisation of 72.

Compare it with how many times it occurs in the factorisations of 12 and 6 put together.

Figure it Out

1. Find the prime factorisations of the following numbers: 64, 104, 105, 243, 320, 141, 1728, 729, 1024, 1331, 1000.
2. The prime factorisation of a number has one 2, two 3s, and one 11. What is the number?
3. Find three prime numbers, all less than 30, whose product is 1955.
4. Find the prime factorisation of these numbers without multiplying first
   a. 56×25 b. 108×75 c. 1000×81
5. What is the smallest number whose prime factorisation has:
   a. three different prime numbers?
   b. four different prime numbers?

Prime factorisation is of fundamental importance in the study of numbers. Let us discuss two ways in which it can be useful.

Using prime factorisation to check if two numbers are co-prime

Let us again take the numbers 56 and 63. How can we check if they are co-prime? We can use the prime factorisation of both numbers—

$$56 = 2 \times 2 \times 2 \times 7 \text{ and } 63 = 3 \times 3 \times 7$$

Now, we see that 7 is a prime factor of 56 as well as 63. Therefore, 56 and 63 are not co-prime.

What about 80 and 63? Their prime factorisations are as follows:

$$80 = 2 \times 2 \times 2 \times 2 \times 5 \text{ and } 63 = 3 \times 3 \times 7$$

There are no common prime factors. Can we conclude that they are co-prime? Suppose they have a common factor that is composite. Would the prime factors of this composite common factor appear in the prime factorisation of 80 and 63?

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Therefore, we can say that if there are no common prime factors, then the two numbers are co-prime.

Let us see some examples.

Example: Consider 40 and 231. Their prime factorisations are as follows:

$$40 = 2 \times 2 \times 2 \times 5 \text{ and } 231 = 3 \times 7 \times 11$$

We see that there are no common primes that divide both 40 and 231. Indeed, the prime factors of 40 are 2 and 5 while, the prime factors of 231 are 3, 7, and 11. Therefore, 40 and 231 are co-prime!

Example: Consider 242 and 195. Their prime factorisations are as follows:

$$242 = 2 \times 11 \times 11 \text{ and } 195 = 3 \times 5 \times 13$$

The prime factors of 242 are 2 and 11. The prime factors of 195 are 3, 5, and 13. There are no common prime factors. Therefore, 242 and 195 are co-prime.

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Using prime factorisation to check if one number is divisible by another

We can say that if one number is divisible by another, the prime factorisation of the second number is included in the prime factorisation of the first number.

We say that 48 is divisible by 12 because when we divide 48 by 12, the remainder is zero. How can we check if one number is divisible by another without carrying out long division?

Example: Is 168 divisible by 12? Find the prime factorisations of both:

$$168 = 2 \times 2 \times 2 \times 3 \times 7 \text{ and } 12 = 2 \times 2 \times 3$$

Since we can multiply in any order, now it is clear that,

$$168 = 2 \times 2 \times 3 \times 2 \times 7 = 12 \times 14$$

Therefore, 168 is divisible by 12.

Example: Is 75 divisible by 21? Find the prime factorisations of both:

$$75 = 3 \times 5 \times 5 \text{ and } 21 = 3 \times 7$$

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As we saw in the discussion above, if 75 was a multiple of 21, then all prime factors of 21 would also be prime factors of 75. However, 7 is a prime factor of 21 but not a prime factor of 75. Therefore, 75 is not divisible by 21.

Example: Is 42 divisible by 12? Find the prime factorisations of both:

$$42 = 2 \times 3 \times 7 \text{ and } 12 = 2 \times 2 \times 3$$

All prime factors of 12 are also prime factors of 42. But the prime factorisation of 12 is not included in the prime factorisation of 42. This is because 2 occurs twice in the prime factorisation of 12 but only once in the prime factorisation of 42. This means that 42 is not divisible by 12.

We can say that if one number is divisible by another, then the prime factorisation of the second number is included in the prime factorisation of the first number.

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Figure it Out

1. Are the following pairs of numbers co-prime? Guess first and then use prime factorisation to verify your answer.
   a. 30 and 45 b. 57 and 85
   c. 121 and 1331 d. 343 and 216
2. Is the first number divisible by the second? Use prime factorisation.
   a. 225 and 27 b. 96 and 24
   c. 343 and 17 d. 999 and 99
3. The first number has prime factorisation $2 \times 3 \times 7$ and the second number has prime factorisation $3 \times 7 \times 11$. Are they co-prime? Does one of them divide the other?
4. Guna says, “Any two prime numbers are co-prime?”. Is he right?

5.5 Divisibility Tests

So far, we have been finding factors of numbers in different contexts, including to determine if a number is prime or not, or if a given pair of numbers is co-prime or not.

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It is easy to find factors of small numbers. How do we find factors of a large number?

Let us take 8560. Does it have any factors from 2 to 10 (2, 3, 4, 5, ..., 9, 10)?

It is easy to check if some of these numbers are factors or not without doing long division. Can you find them?

Divisibility by 10

Let us take 10. Is 8560 divisible by 10? This is another way of asking if 10 is a factor of 8560.

For this, we can look at the pattern in the multiples of 10.

The first few multiples of 10 are: 10, 20, 30, 40, ... Continue this sequence and observe the pattern.

Is 125 a multiple of 10? Will this number appear in the previous sequence? Why or why not?

Can you now answer if 8560 is divisible by 10?

Consider this statement:

Numbers that are divisible by 10 are those that end with '0'. Do you agree?



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Divisibility by 5

The number 5 is another number whose divisibility can easily be checked. How do we do it?

Explore by listing down the multiples: 5, 10, 15, 20, 25, ... What do you observe about these numbers? Do you see a pattern in the last digit?

What is the largest number less than 399 that is divisible by 5? Is 8560 divisible by 5?

Consider this statement:

Numbers that are divisible by 5 are those that end with either a '0' or a '5'. Do you agree?



Divisibility by 2

The first few multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...
What do you observe? Do you see a pattern in the last digit?

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Is 682 divisible by 2? Can we answer this without doing the long division?

Is 8560 divisible by 2? Why or why not?

Consider this statement:

Numbers that are divisible by 2 are those that end with '0', '2', '4, '6' or '8'. Do you agree?



What are all the multiples of 2 between 399 and 411?

Divisibility by 4

Checking if a number is divisible by 4 can also be done easily!

Look at its multiples: 4, 8, 12, 16, 20, 24, 28, 32, ...

Are you able to observe any patterns that can be used? The multiples of 10, 5 and 2 have a pattern in their last digits which we are able to use to check for divisibility. Similarly, can we check if a number is divisible by 4 by looking at the last digit?

It does not work! Look at 12 and 22. They have the same last digit, but 12 is a multiple of 4 while 22 is not. Similarly 14 and 24 have the same last digit, but 14 is not a multiple of 4 while 24 is. Similarly, 16 and 26 or 18 and 28. What this means is that by looking at the last digit, we cannot tell whether a number is a multiple of 4.

Can we answer the question by looking at more digits? Make a list of multiples of 4 between 1 and 200 and search for a pattern.

Find numbers between 330 and 340 that are divisible by 4. Also, find numbers between 1730 and 1740, and 2030 and 2040, that are divisible by 4. What do you observe?

Is 8536 divisible by 4?

Consider these statements:

1. Only the last two digits matter when deciding if a given number is divisible by 4.
2. If the number formed by the last two digits is divisible by 4, then the original number is divisible by 4.
3. If the original number is divisible by 4, then the number formed by the last two digits is divisible by 4.

Do you agree? Why or why not?

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Divisibility by 8

Interestingly, even checking for divisibility by 8 can be simplified. Can the last two digits be used for this?

Find numbers between 120 and 140 that are divisible by 8. Also find numbers between 1120 and 1140, and 3120 and 3140, that are divisible by 8. What do you observe?

Change the last two digits of 8560 so that the resulting number is a multiple of 8.

Consider these statements:

1. Only the last three digits matter when deciding if a given number is divisible by 8.
2. If the number formed by the last three digits is divisible by 8, then the original number is divisible by 8.
3. If the original number is divisible by 8, then the number formed by the last three digits is divisible by 8.

Do you agree? Why or why not?



We have seen that long division is not always needed to check if a number is a factor or not. We have made use of certain observations to come up with simple methods for 10, 5, 2, 4, 8. Do we have such simple methods for other numbers as well? We will discuss simple methods to test divisibility by 3, 6, 7, and 9 in later classes!

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Figure it Out

1. 2024 is a leap year (as February has 29 days). Leap years occur in the years that are multiples of 4, except for those years that are evenly divisible by 100 but not 400.
   a. From the year you were born till now, which years were leap years?
   b. From the year 2024 till 2099, how many leap years are there?
2. Find the largest and smallest 4-digit numbers that are divisible by 4 and are also palindromes.
3. Explore and find out if each statement is always true, sometimes true or never true. You can give examples to support your reasoning.

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a. Sum of two even numbers gives a multiple of 4.
b. Sum of two odd numbers gives a multiple of 4.

4. Find the remainders obtained when each of the following numbers are divided by a) 10, b) 5, c) 2.

78, 99, 173, 572, 980, 1111, 2345

5. The teacher asked if 14560 is divisible by all of 2, 4, 5, 8 and 10. Guna checked for divisibility of 14560 by only two of these numbers and then declared that it was also divisible by all of them. What could those two numbers be?

6. Which of the following numbers are divisible by all of 2, 4, 5, 8 and 10: 572, 2352, 5600, 6000, 77622160.

7. Write two numbers whose product is 10000. The two numbers should not have 0 as the units digit.

5.6 Fun with Numbers

Special numbers

There are four numbers in this box. Which number looks special to you? Why do you say so?

9	16
25	43

Look at the what Guna's classmates have to share:

- Karnawati says, "9 is special because it is a single-digit number whereas all the other numbers are 2-digit numbers".
- Gurupreet says, "9 is special because it is the only number that is a multiple of 3".
- Murugan says, "16 is special because it is the only even number and also the only multiple of 4".
- Gopika says, "25 is special as it is the only multiple of 5".
- Yadnyikee says, "43 is special because it is the only prime number".
- Radha says, "43 is special because it is the only number that is not a square".

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Below are some boxes with four numbers in each box. Within each box try to say how each number is special compared to the rest. Share with your classmates and find out who else gave the same reasons as you did. Did anyone give different reasons that may not have occurred to you?!

Math Talk

5	7
12	35

3	8
11	24

27	3
123	31

17	27
44	65

A prime puzzle

The figure on the left shows the puzzle. The figure on the right shows the solution of the puzzle. Think what the rules can be to solve the puzzle.

Math Talk

			75
			42
			102
170	30	63	

5	5	3	75
2	3	7	42
17	2	3	102
170	30	63	

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Rules

Fill the grid with prime numbers only so that the product of each row is the number to the right of the row and the product of each column is the number below the column.

			105
			20
			30
28	125	18	

			8
			105
			70
30	70	28	

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			63
			27
			190
45	42	171	

			343
			660
			44
28	154	231	

SUMMARY

- If a number is divisible by another, the second number is called a **factor** of the first. For example, 4 is a factor of 12 because 12 is divisible by 4 ($12 \div 4 = 3$).
- **Prime numbers** are numbers like 2, 3, 5, 7, 11, ... that have only two factors, namely 1 and themselves.
- **Composite numbers** are numbers like 4, 6, 8, 9, ... that have more than 2 factors, i.e., at least one factor other than 1 and themselves. For example, 8 has the factor 4 and 9 has the factor 3, so 8 and 9 are both composite.
- Every number greater than 1 can be written as a product of prime numbers. This is called the number's **prime factorisation**. For example, $84 = 2 \times 2 \times 3 \times 7$.
- There is only one way to factorise a number into primes, except for the ordering of the factors.
- Two numbers that do not have a common factor other than 1 are said to be **co-prime**.
- To check if two numbers are co-prime, we can first find their prime factorisations and check if there is a common prime factor. If there is no common prime factor, they are co-prime, and otherwise they are not.
- A number is a factor of another number if the prime factorisation of the first number is included in the prime factorisation of the second number.

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